The Taylor Series

- FOR YOUR NOTEBOOK!
  \[ y(t + \Delta t) = y(t) + \Delta t \frac{dy}{dt}(t) + \frac{\Delta t^2}{2!} \frac{d^2y}{dt^2}(t) + \frac{\Delta t^3}{3!} \frac{d^3y}{dt^3}(t) + \cdots \]

- Get the following... for the 0th through 5th order.
  \[ t_0 = 1, \quad \Delta t = 2 \quad y(3) = ? \]

- Answer: 1, 7, 19, 27, 27, 27.....

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Lecture 4:
A little more Math & Stella Orientation (Intro)

- Exponentials Recap
- Stella Tutorial

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The Exponential

- The "exponential"
  - e
  - 2.71828

- \( \int \frac{dx}{x} = \ln x + C = \log_e x + C \)

- \( e^{\ln x} = x \)

- \( \frac{d}{dx} e^{u(x)} = e^{u(x)} \frac{d}{dx} u(x) \)

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Agenda

- Why we use "e" in analytic solutions.

- We’ll be doing the online help to guide our way through the Stella Interface.

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The Exponential

- The exponential serves to represent a number of physical processes that also satisfy many of the differential equations our problems generate.
  - Exponential Growth
  - Exponential Decay
  - Select Oscillations Processes
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The Exponential (2.71828)

- Exponential Growth
  \[ y(t) = e^{At} \]
  \[ \frac{dy}{dt} = A e^{At} \]
  \[ \frac{dy}{dt} = Ay \]

- Exponential Decay
  \[ y(t) = e^{-At} \]
  \[ \frac{dy}{dt} = -A e^{-At} \]
  \[ \frac{dy}{dt} = -Ay \]

Let’s do that again….

- Exponential Growth
  \[ y(t) = b e^{at} \]
  \[ \frac{dy}{dt} = b a e^{at} \]
  \[ \frac{dy}{dt} = ay \]

The Exponential (2.71828)

Oscillations (Euler’s Formula)

\[ e^x = \cos(x) + i \sin(x) \]

\[ \frac{d^2}{dt^2} U_{ag} = -\omega^2 U_{ag} \]

\[ U_{ag} = U_{ag0} e^{-\omega t} \]

\[ U_{ag} = U_{ag0} \cos(\omega t) - U_{ag0}/\sin(\omega t) \]

\[ U_{ag} = U_{ag0} + V_{ag0}i \]

A Naturally Oscillating System

Predator-Prey Model

- V = Vegetation
- H = Herbivores
- C = Carnivores

\[ \frac{dH}{dt} = \alpha V - \alpha H - \alpha CH \]

\[ \frac{dC}{dt} = \beta (\alpha CH) - \beta C \]

Boundary Layer Meteorology Example

Nocturnal Wind Cycles

- U = Ug + Uag

V = Vegetation
H = Herbivores
C = Carnivores

- Circle of Cartoon Violence (a stable cyclic system)
A Naturally Oscillating System

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Chaotic Oscillations

\[ \frac{dx}{dt} = p(y - x) \]
\[ \frac{dy}{dt} = (r - z)x - y \]
\[ \frac{dz}{dt} = xy - bz \]

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And for the rest of the time...

Stella Play and the Leaky Bucket!
(And if you want, Earth’s energy budget)
Continues into Monday...

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