Lab Follow-up

The Bucket

\[ q_{in} = q_{in} \quad \{m^3 s^{-1}\} \]
\[ q_{out} = A_h (2gh) \quad \{m^3 s^{-1}\} \]
\[ q_{out} = \pi r_h^2 2g \left( \frac{V}{\pi r_b^2} \right)^{1/2} \]
\[ \frac{dV}{dt} = q_{in} - q_{out} \]
\[ \frac{dV}{dt} = \frac{q_{in}}{\alpha} \left(2g\pi\right)^{1/2} \left(\frac{r_h^2}{r_b}\right)^{\beta} \sqrt{V} \]
\[ \frac{dV}{dt} = \alpha - \beta V^{1/2} \]
Lab Follow-up

- Look again!

\[ \frac{dV}{dt} = q_{in} - (2g\pi)^{\frac{1}{2}} \left( \frac{r_h^2}{r_b^2} \right) V^{\frac{3}{2}} \]

\[ \frac{dV}{dt} = \alpha - \beta V^{\frac{3}{2}} \]

- What happens when
  - \( r_h \rightarrow r_b \)?
  - \( r_h > r_b \)?
The “Danger to Yourself and Others” Award!

- Look again!

\[
\frac{dV}{dt} = q_{\text{in}} - (2g\pi)^{\frac{1}{2}} \left( \frac{r_h^2}{r_b} \right) V^{\frac{1}{2}}
\]

\[
\frac{dV}{dt} = \alpha - \beta V^{\frac{1}{2}}
\]

- What happens when
  - \( r_h \to r_b \)?
  - \( r_h > r_b \)?
Always remember the Modeling Context!

- We therefore must assign a physical constraint

\[
\frac{dV}{dt} = \frac{q_{in}}{\alpha} - (2g\pi)^\frac{1}{2} \left( \frac{r_h^2}{r_b} \right)^\beta \sqrt{V}
\]

- Surprisingly, it still gives you an “answer!”
Always remember the Modeling Context!

- Like many models, this one has limits of applicability.

\[
\frac{dV}{dt} = \frac{q_{in}}{\alpha} - \left(2g\pi\right)^{\frac{1}{2}} \left(\frac{r_h^2}{r_b}\right)^{\beta} r \sqrt{V}
\]

where \( r_h \ll r_b \)
Lecture 8: Water Flows in the Mono Basin

An example of a physical modeling scenario

Also... Variable “Converters” Graphs and Equations.
The Mono Lake Case

- Chapter Readings
  - Cadillac Desert (Book or PBS Series)
  - This case provides a simple example of modeling a real-world scenario
The Skinny

LA Gets a bulk of its water from the Mono Lake Basin (along the Northern California/Nevada Border) since 1940's.
The Background

- http://www.wsu.edu/~forda/mb.html
- LA Gets a bulk of its water from the Mono Lake Basin (along the Northern California/Nevada Border) since 1940’s.
- LA Has a Lot of Swimming Pools and Movie Stars.
- Therefore LA Needs a lot of water.
The Skinny

- The Water has to come from somewhere (Hint. It’s not going to be Orange County or San Diego.)
- It’s going to come from a place where $P$ should be greater than $E$.
- The Lake is also a valued ecosystem with critical watch points.
- http://www.wsu.edu/~forda/EXX.htm
Starting Points

- Pen & Paper!
  - List primary stock(s)
    - Lake Volume
    - Supplementary Reservoirs and Catchments, etc.
  - List primary sources and Sinks.
    - Streamflow in and out of the lake
    - Overland Flow into lake
    - Precip and Evaporation
Starting Points

- Pen & Paper!
  - List dependant variables on the sources and sinks
    - Precip Rate
    - Potential Evaporation Rate
    - Gaged Inflow
    - Estimates of Miscellaneous Net Flows
    - Lake Area vs Volume
  - NOW. Make a point to notice what parameters can be represented explicitly and which must be approximated or “parameterized”
Lay down the test system

- Create a framework by which you can do a pilot simulation by which the basics of the system are laid down.
  - This system will not be the complete system by far.
  - However, it should provide enough latitude for future changes as you add more and more complexity.
- In programming a modular approach works best here.
The Reservoir and Primary Sources and Sinks

- Volume
  - Sinks and Sources should be grouped by processes ($L^3/T$)
  - Gaged {streamflow} Inflows
  - Precipitation
  - Free Evaporation
  - [Evapotranspiration]
  - Miscellaneous Inflows and Outflows
What we aren’t going to include (for now!)

- The Volume/Area relationship
  - Lakes are Bowls.

- We are going to use a simple bucket to start with

- The V/A/depth problem will be addressed shortly
Stella Loading

- Construct the basics of the system in Stella
  - Surface Area ($A$) = 30 K-acres (temporary!)
  - Precip & Evap ($P$ & $E_p$) = 0.67 ft/yr and 3.75 ft/yr
  - Gaged Flow ($S_g$) = 150 K-acre feet/yr
  - Swimming Pools, Movie Stars ($D_e$) = 100 KAF/yr
  - Misc Inflow = 47.6 KAF/yr
  - Ungaged Inflow ($S_m$) & Municipal Diversions ($D_\ell$)
  - Misc Outflows = 33.6 KAF/yr
  - Dry-Evap & ET ($ET$ for both)
  - Now, what does this look mathematically?
The Math

- Easy enough...
  - Always identify all your variables explicitly though.

\[
\frac{dV}{dt} = P + (S_g - D_e) + (S_m - D_l) - (E_p + ET)
\]

- You should be able to do this “freehand” as you get better at this!
Now do the basic model

Off to Stella...
The next layer of complexity

- The lake Volume vs Depth vs Area
  - Precipitation inflow and Evaporation outflow is not from a Volume – it’s from an AREA!

- This can be done a few ways.
  - A lookup table – as done in the book
  - A regression curve based on the table
Tables vs Regressions

\[ A(V) = -2.021 \times 10^{-14} V^4 + 5.478 \times 10^{-10} V^3 - 5.462 \times 10^{-6} V^2 + 2.766 \times 10^{-2} V + 3.535 \times 10^{-1} \]

\[ Z(V) = -2.134 \times 10^{-13} V^4 + 4.622 \times 10^{-9} V^3 - 3.513 \times 10^{-5} V^2 + 1.263 \times 10^{-1} V + 6.228 \times 10^3 \]
Advantage of the Table

- Readily editable.
- Possibly better than a regression if the $Y(x)$ relationship is quirky or otherwise a challenge to match as you go to increasingly complex regression formulae.
Advantage of the Regression

- If Smooth \emph{and} a Good Fit \emph{and} a match for the system, the regression...
  - Mathematically more seamless than a more jerky table. And better, numerically, if it’s monotonic (always goes up, or always goes down)

\[
\frac{dV}{dt} \bigg|_{P-E} = (P - E)A
\]

where \( A = f(V) \)
Which to use?

Whichever one works best!
Surface Hydrology Dam Component

- Sheridan Lake: Unregulated Reservoir
  - Runge-Kutta Method Applied to Close Storage and Reservoir Elevation Relationships

\[
\frac{\Delta S}{\Delta t} = \ln_{str}(t) + \ln_{sfc}(t) - \text{Out}(H)
\]

\[
\frac{\Delta H}{\Delta t} = \frac{\ln_{total}(t) - \text{Out}(H)}{\text{Area}(H)}
\]

Your Prof likes SMOOTH solutions wherever possible…
Some new stuff for many of you

- **The [A]MOD function**
  - I use this a lot in climate modeling

- **The Graph Feature in Stella**
  - Not your prof’s favorite feature

- **Specific Gravity**

  \[ sg = \frac{\rho_w \sqrt{V_w} + m_s}{\rho_w \sqrt{V}} \]

  \[ sg = \frac{(1.36 \frac{t}{kAc - ft})(2228 \frac{kAc}{ft}) + (230 \frac{t}{kAc - ft})}{(1.36 \frac{t}{kAc - ft})(2228 \frac{kAc}{ft})} \]

- **Hydrology Units**
Then add further complexities

- Variable Precip, Evap and other parameters (external forcings!)
- Separate Miscellaneous Terms

- For way down the path...
  - Coupling with other behavioral models.
  - Link Mono Lake Volumes to an Ecological Model
Deliverables! (Next Thursday*)

- From the Handout Chapter
- Construct, incrementally, the full bells-and-whistle model
  - Problem 1 ("deluxe" plumbing and checks)
  - Problem 2 (dynamic equilibrium)
- Then Play!
  - Problem 3
  - Problem 4
  - TRY!!! PROBLEM 5 (features the (A)MOD function) and beyond...
  - You should be increasingly comfortable doing this entire chapter.