14 Feb 2007 ATM 515: Intro Env Modeling 1

Dr. Capehart Hates Elmo.

- From Last time, we had a Stella model of a 3-player Lotka-Volterra Set.
  \[
  \begin{align*}
  \frac{dP}{dt} &= a - (b + c)P \\
  \frac{dH}{dt} &= (dP - fC - e)H \\
  \frac{dC}{dt} &= (gH - h)C
  \end{align*}
  \]

Dr. Capehart REALLY Hates Elmo.

- This set converges to at least three steady state solutions.
  \[
  \begin{align*}
  \frac{dP}{dt} &= a - (b + c)P \\
  \frac{dH}{dt} &= (dP - fC - e)H \\
  \frac{dC}{dt} &= (gH - h)C
  \end{align*}
  \]

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- Plants Only
- Plants and Bunnies
- All three...
  \[
  \begin{align*}
  \frac{dP}{dt} &= a - (b + c)P \\
  \frac{dH}{dt} &= (dP - fC - e)H \\
  \frac{dC}{dt} &= (gH - h)C
  \end{align*}
  \]

Die Elmo Die!

- We should be calculate steady state values...
  \[
  \begin{align*}
  \frac{dP}{dt} &= a - (b + c)P \\
  \frac{dH}{dt} &= (dP - fC - e)H \\
  \frac{dC}{dt} &= (gH - h)C
  \end{align*}
  \]

No really, Elmo, Just DIE.

- And it’s a lot easier to do this on paper.
  \[
  \begin{align*}
  \frac{dP}{dt} &= a - (b + c)P \\
  \frac{dH}{dt} &= (dP - fC - e)H \\
  \frac{dC}{dt} &= (gH - h)C
  \end{align*}
  \]

H+P (C=0)

- Let’s solve this three ways...
  \[
  \begin{align*}
  \frac{dP}{dt} &= a - (b + c)P \\
  \frac{dH}{dt} &= (dP - e)H \\
  \frac{dC}{dt} &= (gH - h)C
  \end{align*}
  \]
Lecture 9: Competition Modeling

Self-Limiting Growth and the Logistics Equation
Or.. Modeling Good old-fashioned animosity!
And...
Predator-Prey: With an Edge.

From last time

“Nal Komerex Khesterex”
- “That which does not grow, shall wither and die.”

- And sometimes, it helps to grow faster than your neighbor!
  - AKA If you can’t eat ’em, OUT eat ’em

Previous Simulations

- Non Realistic Food Sources
- No completion for Resources.
- Net Growth rates are a function of encounters with separate species

- We have yet to officially recognize that a given species must operate in a resource-limited environment
- Competition Arises.
A new set of scenarios

- Consider a petri dish, field or pond.
  - A "fixed" volume or domain
  - Optimal growth rate and decay rates are set for a specific species of entity
  - Growth is dependent on a lack of competition for resources.
- The greater the area covered (or population) the lower the birth rate.
- NOTEBOOK EXERCISE!
  - What Physical (non biological) systems replicate this trait?

In Stella

\[ \frac{dA}{dt} = g_o f_o A - d_o A \]

The Result where \( n = 1 \) (Linear Growth Response)

Other values of \( n = 2 \) (slow response to crowding)

Other values of \( n = \frac{1}{2} \) (fast response to growth)
How did I get those final numbers?

- The resulting "steady state" value of $f$
  - represents not the maximum possible area of play but the maximum "holding capacity" of player that they system can maintain
  - $f = \frac{d}{K}(1 - \frac{d}{K}) = (1 - \frac{d_{20\%}}{20\%})$

For a General Case

- The formula can be written with respect to a derivative parameter from the steady state solution called the "Carrying Capacity" ($K$) and by a net base growth competition-free net growth/decay rate ($r$) and where $n = 1$ otherwise.
- $K$ is NOT the maximum possible area! Rather it is the maximum holding capacity:
  - Very Bad analogy*: "Field Capacity".

The Logistic Equation

- This then solves to the form to the right and has been a workhorse in population biology
- Hastings(1985)
  - $dA = \frac{rA(K-A)}{K} dt$
  - $A(t) = \frac{A_0 e^{rt}}{1 + \frac{A_0}{K}(e^{rt} - 1)}$

The Logistic Equation

- Alternate Notation for populations ($N$)
- Verhulst, 1838
- $r = (\text{Competition-Free Birth-Death rates})$; notice that it's "game over" if $r < 0$
- $N_0 = \text{initial pop.}$
- $K = \text{maximum holding capacity}$
  - $N(t) = \frac{N_0 e^{rt}}{N_0 + (K - N_0)e^{rt}}$

A Sales Scenario
New Predator Prey Scenario: “Thumper’s Gone Bad!”

☐ Given this, we can have some gangland fun with our rabbits.
☐ Let’s have two gangs of rival rabbits vying for dominance over Farmer Palmer’s Patch. (Sharks and Jets)
☐ Warning, though named parameters are clearly facetious, they can be tied to the same processes we’ve dealt with all along.

Thumper’s Gone Bad!
“If you can’t say something nice, say something nasty!”

☐ Two rival groups
  ☐ Both survive off of the same resource.
  ☐ Resource is inexhaustible
  ☐ No West Side Story
  ☐ (The Sharks mate with Sharks, Jets with Jets)
  ☐ Interactions are fatal
  ☐ No Cannibalism or Highlander Action
  ☐ Music and Dancing optional.
  ☐ NEW: This is competition with an edge.
  ☐ Individual Sharks and Jets compete against each other for resources as well.

In Odum’s Conceptual Landscape

One thing that will make this easier to grasp is that we have two species with similar needs (albeit divergent proclivities!)
Notice that we are spinning “Rivalry” as an active aggression (not a drag on new births). We could do this the other way and integrate it into the source terms.
This would depend on your rules.
Also notice that we don’t have an explicit “natural” demise of our players. We’ll address that later.

In the Stella Variant

Initial Conditions

☐ 3 Inexhaustible Units of Carrots
☐ 3 Units of Sharks
☐ 3 Units of Jets

Now onto the rates

Concretizing the Rates

☐ Shark Love Rate
  ☐ 7.0% (per shark-carrot-year)
☐ Shark Rivalry Rate
  ☐ 0.2% (per shark-shark-year)
☐ Shark Casualty Rate
  ☐ 0.2% (per shark-jet-year)
☐ Sharks are Wimps but Boring and uncooperative

☐ Jet Love Rate
  ☐ 8.0% (per shark-carrot-year)
☐ Jet Rivalry Rate
  ☐ 0.1% (per shark-shark-year)
☐ Jet Casualty Rate
  ☐ 0.1% (per shark-jet-year)
☐ Jets are Mean but Passionate and Organized
Kormorex or Khesterex?

- We now have a much more complicated system.
- Two teams, and self & cross-species competition.
- One side will be winners, one will be losers, but who will be who?

The Sharks are hardwired to fail.
- Not "Prolific"
- More Internally Hostile
- Could also be spun as resource hogs
- Wimps in Battle (heavier casualties)
- The Jets are in a better position
- More "Prolific"
- Less Internally Uncooperative (not more cooperative – notice the sign!)
- Or more frugal with resources
- Superior in Battle (lighter casualties)

Can we turn the tide?  
Hint: you CAN get a steady state with this system!  
Let’s put this one in equation form!

The Math

- Compare and contrast this to our original case (linear example)

K is the ratio of net-passion (r) to rivalry (ρ) in this scenario
- Let C=3
  \[ K = \frac{C}{\beta} \]
  \[ K = 0.08 \]
  \[ K = 0.001 \]
  \[ K = 2400 \]
K=240?

- Let’s taste the pudding
- Surprise! It works!!

What about the Sharks?

- Let’s take away the Jets and insert the Sharks’ “numbers”

\[ K = \frac{C}{P} \]

\[ K = \frac{(0.07)3}{0.002} \]

\[ K = 105 \]

So...

- We have shown that you can create a competitive and complex system that can be predicted both via Stella and without, using a the Logistic Equation as a model
- We’ve extracting the Logistic Equation from a “free-form” scenario.
  - That last one is a very important skill!

Self-Guided Recreational Conflict Resolution

- What would happen if you made the carrot layer variable (as in the previous Wabbit Scenario?)
- Could you calculate a rivalry, passion or toughness parameter to give the Sharks a fighting chance against the Jets?
- But this one may be more fun...

Self-Guided Recreational Mayhem

- Can you create an example with Fudds™ picking the Bunnies (of both factions)?
  - Example Set of Rules
    - All Bunnies are equally Tasty (or not!)
    - Not all Bunnies are available at all times
      - Could one type of Bunny be easier to kill?
    - You can have Blasé or Rival Fudds
    - Happy Wabbit Hunting!

Self-Guided Recreational Mayhem

- But more importantly...
  - What systems (physical/biological/mixed) can you see working with the Logistics paradigm?
Caveat

In all of these, you'll need to be very careful when you model “apples” and “oranges” especially when creating the areas where an item is being consumed (as opposed to just being eliminated)