Lecture 15
Introduction to Chaos

Chaos in Modeling
Agenda

- Introduction to Chaos
- How Chaos Works
- How Chaos can work for you!
  - Ensemble Modeling Concept
Predictability Limits in Modeling

- In all models, errors expand from initialization onwards.*
- Errors can double every 2-3 days limiting most forecasts [*marginal efficacy*] to no more than a couple of weeks.
- Modeling Short waves go first (obviously!) with skill for the longest of waves lasting no more than 3-4 weeks.
... The Predictability Problem

- Even with a “Perfect Model,” “Perfect Numerics,” and “no errors in observations,” there will remain the problem of incomplete analyses (there is always something between point A and B). These erroneous initial fields will always start a set of magnifying errors.

- Example, an unresolvable storm is triggered altering the local mass field, subsequently affecting the local momentum field that feeds upward to the mesoscale and synoptic environments.
Intrinsic Predictability Across Regimes

- Forecasts based on climatology will have a relatively high level of error, but will remain constant over time.
- Forecasts based on persistence (i.e., whatever is happening now will happen later) are nearly perfect at extremely short range, but quickly deteriorate.
- Current models do well at short ranges, but eventually do worse than climatology.
- A forecast that is worse than climatology is considered useless.
Why is there a predictability limit

- Non-Linearities in Physical Systems can amplify deviations of an *ensemble* of simulation results from sets of *near identical* initial conditions

“Chaos Theory”

Ref: Lorenz, 1993: The Essence of Chaos, UW Press.
Chaos’s Heritage

Based in Determinism

- A system can be understood and predicted based on [quantifiable] laws and good data
- However a complex (non-linear) system can be highly sensitive to apparently trivial and negligible changes in initial conditions
- In general, one could assert that a long-term simulation initialized from near-precise initial conditions has no more skill than a random initialization
- Chaos is not a refutation of determinism, but in inevitable attribute of determinism plus a lack of precision in observations
Lorenz and Chaos

- Lorenz’s models demonstrated Chaos-based unpredictability in atmospheric systems.

- Rather than being an enemy of prediction, Chaos has been demonstrated to be an ally (or a canary) in NWP and its forecasting.
Example of [Un]Predictability

- Take a ski slope, pachinko machine, etc
Riding the Fall Line

Dx = 10cm

Dx = 1mm
State-Space and Attractors

Plotting variables that represent our system in state-space (as opposed to physical space, for example) can demonstrate that seemingly quasi-chaotic system has a sense of organization.

X vs V state space
From the Slopes to the Sky

- As indicated before even a perfect model can give an imperfect forecast
  - Incomplete Data
  - Faulty Analyses
- “That Butterfly Sound Byte”
  - Small perturbations [deviations from a given/mean state] propagate to larger scale systems.
- Can this lack of predictability be “exploited”?
  - The Ensemble Forecast Battery
    - December 1997, *Weather and Forecasting*
    - *Sivillo et al.*
Chaos in NWP

Given a set of initial conditions, the predictability of a system rests inside of a finite envelope towards the beginning of the simulation.
Stella™ model of a Simplistic Atmospheric System

Midlatitude cyclone

Baroclinic Drag

Slope Delta T

Delta y

T0

Cold damping

Advection rate

Growth rate

-47,775,849

South avg accum

South avg

North avg accum

North avg

South avg out

North avg out

West

East

T warm 0

T cold 0

DT Dy

DT Dz

Delta y

T cold 0

Cold recharge

Recharge rate

Warm recharge

Advection rate

Recharge rate

Warm damping

Drag coefficient

South avg out

North avg out

T0

8.10

8.80

8.90

8.20

Growth rate

8.20

-47,775,849
Stella™ model of a Simplistic Thermally-driven system System

Perhaps, you’d prefer (and remember) this?
Or when boiled down…

\[
\frac{dx}{dt} = p(y - x)
\]

\[
\frac{dy}{dt} = (r - z)x - y
\]

\[
\frac{dz}{dt} = xy - bz
\]
Or when boiled down...

\[ \frac{dx}{dt} = p(y - x) \]
\[ \frac{dy}{dt} = (r - z)x - y \]
\[ \frac{dz}{dt} = xy - bz \]

Given

\[ p \cdot (y - x) = 0 \]
\[ (r - z) \cdot x - y = 0 \]
\[ x \cdot y - b \cdot z = 0 \]

Find \((x, y, z) \rightarrow \)

\[
\begin{bmatrix}
1 & 1 \\
0 & (b \cdot r - b)^2 & -(b \cdot r - b)^2 \\
0 & (b \cdot r - b)^2 & -(b \cdot r - b)^2 \\
0 & r - 1 & r - 1
\end{bmatrix}
\]
\[
\frac{dx}{dt} = p(y - x) \\
\frac{dy}{dt} = (r - z)x - y \\
\frac{dz}{dt} = xy - bz
\]
"...& doesn't look like what we're used to!"
Chaos in a Forecast Ensemble

- Each Color = 1 231-day run
- Initial Northern-most temperature perturbed $\theta_N \pm \Delta \theta$
Stella Model on F: Drives

Models Housed at
- F:\Dept\IAS\ATM 515\ATM 550_Midlatitude cyclone.stm
- F:\Dept\IAS\ATM 515\ATM 550 Simplified Lorenz Chaos.stm

Executable From
- F:\Dept\IAS\Stella7b\Stella7b.exe
Ensemble State-spaces

Like the sled example, multiple forecasted variables can be plotted against each other to illustrate attractor principles.
The Ensemble Forecast

Goal:

- To create a collection of simulations of a given case representing quantifiably probable outcomes.
- To assess the reliability/robustness of a given model.
- Identify regions where special data acquisition is acquired.
Requisite Knowledge for a EsFxB

- The proper set of initial conditions for each battery
- Number of required forecasts in a battery
- How to extract a forecast product from the battery
- How to present the product to the forecasters!
- How should the forecaster interpret the ensemble!
Constructing the Batteries (Init Conds)

- \#sims < actual possible configurations! Due to the need to generate an operational forecast product.
- One approach: Choose IC’s by perturbing parameters that lead to the errors.
  - Chose perturbations that would grow the most during the a given phase on the forecast (assuming that all variations are almost equally likely)
  - Using previous forecast errors as guidance for likely perturbations
- For regional models one must also include boundary conditions when assembling a battery
Getting Number of Batteries

- Primary Restriction: Operational Time Constraints
- Recall also that high-res models require longer clock time!
- Accurate prediction of small waves is the first to go in extended forecasts!
- Revelation: Due to predictability an extended high resolution forecast may be no more accurate than a coarse resolution one!
Extracting the Information
- Weighted Averages based on previous long term forecast errors.

Generating the Displays
- Weighted means, w/ Variations & Probabilities
- Spaghetti Diagrams

Interpretation of Results
- Application example: Regions of consistent high variability
The MRF/GFS Ensemble

URL:
http://wwwt.emc.ncep.noaa.gov/gmb/ens/index.html
Experimental ETA/RSM Ensemble

URL: http://wwwt.emc.ncep.noaa.gov/mmb/SREF/SREF.html
Means and Spreads

12Z Thursday 25 Ja 01

12Z Monday 29 Ja 01
Means and Probabilities

Relative measure of predictability (colors) for ensemble mean forecast (contours) of 500 hPa height

00Z Friday 25 Ja 01

12Z Friday 09 Fe 01
Spaghetti Diagrams

URL: http://www.cdc.noaa.gov/map/images/ens/ens.html
More Spaghetti

URL: http://www.cdc.noaa.gov/map/images/ens/ens.html