Section 6. Time-Dependent Volume Changes of Concrete: Shrinkage and Creep

Hardened concrete changes volume with time. The magnitude and rate of the change depend on the relative humidity of the environment and the applied stress. The changes are easier to understand if we consider them in only one direction and in an environment of constant stress and relative humidity.

Consider a concrete cylinder immediately after the concrete has set. The axial stress caused by selfweight may be ignored. In a dry environment, the height of the cylinder will shorten with time (as will the diameter). This shortening is attributed primarily to loss of adsorbed water. The loss sets up internal stresses that lead to compressive strain or shortening of the cylinder. The change in strain is higher near the surfaces of the cylinder. If we ignore that and plot the ratio of the shortening of the cylinder against time, the relationship observed is described qualitatively by Fig. 6-1. The shortening is rapid initially but slows down with time. In an environment of constant moderate humidity, much of it occurs within three months. Shortening of the cylinder without externally applied force is called shrinkage.

Consider the same cylinder with an applied axial force at an arbitrary age t₀. Assuming that the applied force generates a compressive stress, f_c, below half the strength of the concrete, the measured change in strain would be approximately \( \varepsilon_c = f_c / E_c \), where \( E_c \) is the Young’s modulus. If the force is sustained and strain measurements are continued beyond time t₀, the curve to be obtained, with the shrinkage strain subtracted from the results, will look like the one in Fig. 6-2. The portion of the time-dependent strain related to applied stress is called creep strain. It varies with

![Figure 6-1. Variation of Shrinkage Strain over Time](image1)

![Figure 6-2. Variation of Creep Strain over Time](image2)
time in a manner similar to the variation of shrinkage but it starts at a different time. It is usually expressed as a multiple of the instantaneous strain, $\varepsilon_i$.

Both shrinkage and creep depend on many factors. What we know about them comes from experience and experiments. In the following, we shall refer to the approach adopted by the European Concrete Committee to note the main factors that affect the rate and magnitude of volume changes attributed to loss of adsorbed water.

As we discuss the numbers offered by the European Committee, the student should keep in mind that the best source of information for a structural engineer is the observed behavior of concrete in buildings built using the same materials and under similar environmental conditions as the one she/he intends to build. If the student is ever required to estimate volumetric changes of concrete, she/he should remember that, in general, shrinkage unit strain is on the order of 0.0004 and the additional strain caused by creep is from one to three times the instantaneous strain. Seventy-five percent of the total expected change occurs within one year after setting for shrinkage and one year after loading for creep.

**Shrinkage**

The main variables affecting shrinkage are:

- Humidity
- Volume to surface ratio (the shape of the cross section of the structural element)
- Amount of cement
- Water/cement ratio

Figure 6-3 shows the variation of total shrinkage strain ($\varepsilon_{sh}$) with humidity. The data plotted were obtained for:

- plain concrete made with 600 lb to 750 lb of cement per yd$^3$
- concrete at room temperature
- water/cement ratio of approximately 0.45 (shrinkage increases with increasing water-cement ratio, shrinkage strains for a water/cement ratio of 0.6 being close to 50% higher than those given)

The reader should realize that the ranges within which the data shown are applicable are narrow. And yet, within those ranges, the spread of the data is large. For, say, a relative humidity of 70%, the mean estimate of shrinkage strain is approximately 0.00025 but, even if we ignore extreme values, we observe that the shrinkage strain can be as high as 0.0004 and as low as 0.0001.
Shrinkage can be controlled to be low by using the proper admixture and curing process.

### Creep

Creep is quantified as the ratio of additional strain (caused by creep) to instantaneous strain $\varepsilon_1$ at stress levels below half the compressive strength. This ratio is called “creep coefficient” $\phi_0$ and it too is sensitive to relative humidity (Fig. 6-4).

We note that the mean value of the creep coefficient varies from approximately 1 to approximately 2.5. At 70% relative humidity (within the 5- to 95-percentile limits) it ranges from 1¼ to 2.

Creep varies with thickness and water to cement ratio, as in the case for shrinkage. The values shown in Fig. 6-4 apply to:

- plain concrete made with 600 to 750 lb of cement per cubic yard and loaded 2 to 6 weeks after it is cast
- concrete at room temperature
- ratio of volume to surface approximately equal to 6 in. (the creep coefficient increases with decreasing ratio of volume to surface, creep coefficients at a volume to surface ratio of 3 in. being approximately 6/5 of those given),
- water-cement ratio of approximately 0.45 (creep increases with increasing water-cement ratio, creep strains for a water cement ratio of 0.6 being close to 60% higher than those given)

The important conclusion we make about the creep strain is that it too is affected by the relative humidity, shape of the concrete element, the cement content, and the water/cement ratio. In addition, its magnitude is sensitive to age at loading. Usually, the creep coefficient does not exceed 3.

It is also important to know that, although it is proportional to the initial applied stress, the additional strains caused by creep do not disappear when the stress is removed. This is illustrated in Figure 6-5. The total deformation in an element loaded over a long period of time is the product of the initial deformation and $(1+\varphi_0)$. When the element is unloaded, part of the total deformation is recovered immediately, another part is recovered over time, and the rest never disappears.

![Figure 6-5. Variation of Deformations in Concrete over Time.](image)
**Shrinkage and Creep vs. Time**

The rate of change with time of shrinkage and creep is represented by a coefficient $\rho_t$, which indicates the strain at a given time as a fraction of the total strain.

It is interesting to observe that, according to the experience summarized in Fig. 6-6, one cannot make the claim that the time-dependent volume change ever stops. We also note that almost 90% of the time-dependent strain should occur within the first two years (if there is no strong change in humidity and, in the case of creep, in load).

**Example 1**

A 12-ft long column with a 22 in. x 22 in. cross section supports a load of 200 kips. Estimate the order of magnitude of the total shortening of the column after five years. Assume $E_c = 4,000$ksi.

**Solution**

The instantaneous strain is:

$$\varepsilon_o = \frac{\sigma}{E_c} = \frac{P/A}{E_c} = \frac{200 \text{kip} / (22 \text{in.} \times 22 \text{in.})}{4000 \text{ksi}} \approx 0.0001$$

We know that the shrinkage strain is not likely to exceed 0.0004 by much, so we assume:

$$\varepsilon_{sh} = 0.0004$$

The creep coefficient is not likely to exceed:

$$\phi_o = 3$$

And after 5 years most of the long term deformations are likely to have taken place ($\rho_t = 1$):

$$\rho_t = \varepsilon_o + \rho \phi (\varepsilon_{sh} + \phi \varepsilon \varepsilon_o) = 0.0001 + 1 \times (0.0004 + 3 \times 0.0001) = 0.0008.$$  

The total shortening is therefore likely to be on the order of: $\delta = 0.0008 \times 144 \text{ in.} = \sim 1/8 \text{ in.}$

**Example 2**

The two-story parking lot structure shown is to be built monolithically with 4000 psi concrete. The slabs are 12-in. thick and columns have 20x20-in. square cross sections. We are asked to
estimate the forces induced by shrinkage in columns B1, B2, and C1 (numbers and letters refer to column lines). Assume that the stiffness of the slab is much larger than the stiffness of the columns. Assume the columns do not crack (or remain elastic).

**Solution**

We know that the shrinkage strain is not likely to be much larger than 0.0004. We use this value to estimate the lateral movement of the columns at the level of the floors. Assuming that both slabs shrink the same amount and that shrinkage is uniform through each slab, we compute the displacement at each floor level to be:

For column B1: \( \delta = 0.0004 \times 30 \text{ft} \times 12 \text{in./ft} = 0.14 \text{in} \). If both slabs shrink the same amount, all of this deformation is concentrated in the first story. Column B2, being in the center, would not move if shrinkage is uniform. Column C1 moves more because the distance form it to the center (the point we assume not to move) is larger: \( 0.0004 \times 30 \text{ft} \times 50 \times 12 \text{in./ft} = 0.2 \text{in} \).

We are asked to assume the slabs to be much stiffer than the columns. We can therefore assume the bending moment diagram to be as shown in Fig. 6-8 and the shear force in each column to be:

\[
V = K \delta \\
\text{where } K = \frac{12E/I_g}{H} = \frac{12 \times 3,600 \text{ksi} \times \frac{20 \text{in.} \times (20 \text{in.})^3}{144 \text{in.} \times 12 \text{in.}^3}} = 250 \text{kip/in.}
\]

\( H \) is the clear height of the column (equal to the story height minus the slab thickness). \( I_g \) is the moment of inertia of the cross section. Observe that displacements in Column B1 take place along one of the principal axes of its section. Column C1, on the other hand, deforms along one of the diagonals of its cross section. The moment of inertia \( I_g \) is computed about different axes for B1 and C1. The result, however, is numerically equal. That is, the moment of inertia of a square with respect to one of its principal axes is equal to its moment of inertia with respect to one of its diagonals.

The estimated force induced in B1 is therefore: 250 kip/in \( \times 0.14 \text{ in} = 35 \text{ kip} \).

The estimated force induced in C1 is: 250 kip/in \( \times 0.20 \text{ in} = 50 \text{ kip} \).

These forces would take place in the first story. No forces would be induced in columns in the second story.

**Exercise**

Repeat Example 1 assuming that we do not remove the shoring very soon.

Repeat Example 2 assuming the curing period is long enough.