Section 10 Questions

1) Repeat the example assuming the area of steel is increased to $2A_s$.

An 18ft long beam with an 18"x9" cross section is subjected to constant moment. The beam is reinforced with a single layer of Grade 60 reinforcing bars with the total area given. The bars are located at a depth of $d = 15"$ from the extreme fiber in compression. Estimate the unit strain and the unit stress in the reinforcement assuming $\theta = 2$ deg. Assumed the depth to the neutral axis is $d/3 = 5"$.

Given:

- $b := 9\text{in}$ width of beam
- $h := 18\text{in}$ height of beam
- $d := 15\text{in}$ depth to reinforcement
- $c := \frac{d}{3}$ depth to the neutral axis
- $L := 18\text{ft}$ length of beam
- $\Delta x := 9\text{ft}$ distance over which the angle of curvature occurs
- $\Delta \theta := 2\text{deg}$ angle of rotation of beam
- $f_y := 60\text{ksi}$ yield stress of reinforcement
- $E_s := 29\cdot10^3\text{ksi}$ young's modulus for steel
**Solution:**

Through calculations, one will find that if the neutral axis does not move, then the answers remain the same.

Yield strain of reinforcement

\[
\varepsilon_y := \frac{f_y}{E_s} \quad \varepsilon_y = 2.1 \times 10^{-3}
\]

Curvature

\[
\varphi = \frac{\Delta \theta}{\Delta x}
\]

\[
\varphi := \frac{\Delta \theta}{\Delta x} \quad \varphi = 3.2 \times 10^{-4} \cdot \text{in}
\]

Unit strain at reinforcement

\[
\varepsilon_s := \varphi (d - c) \quad \varepsilon_s = 3.2 \times 10^{-3}
\]

Unit stress in the reinforcement

Since \(\varepsilon_s > \varepsilon_y\), it is likely the steel has yielded. Therefore, the stress in the reinforcement is 60ksi.

\[f_y = 60 \cdot \text{ksi}\]

**2) Compute the unit strain in the reinforcement and the maximum compressive strain in the concrete assuming \(c = d/4\) and \(c = d/5\). (d and \(\Phi\) are the same from exercise 1)**

depths to neutral axis

\[
c_1 := \frac{d}{4} \quad c_1 = 3.8 \cdot \text{in}
\]

\[
c_2 := \frac{d}{5} \quad c_2 = 3 \cdot \text{in}
\]

strains in the reinforcement

\[
\varepsilon_{s1} := \varphi (d - c_1) \quad \varepsilon_{s1} = 0.0036
\]

\[
\varepsilon_{s2} := \varphi (d - c_2) \quad \varepsilon_{s2} = 0.0039
\]

maximum compressive strains in the concrete

\[
\varepsilon_{c1} := \varphi c_1 \quad \varepsilon_{c1} = 0.0012
\]

\[
\varepsilon_{c2} := \varphi c_2 \quad \varepsilon_{c2} = 0.00097
\]
Section 11 Questions

For the conditions describes in the example:
For the reinforced concrete beam shown, \( L = 24\text{ft}, P = 40\text{kip}, \ d = 24\text{in}. \)
Assume all internal tensile forces are resisted by the reinforcement. Assume that the resultant of compressive forces acts at a distance of \( 0.1d \) from the top face of the beam. Assume the total cross-sectional area of reinforcing bars is \( A_s = 3\text{in}^2 \). Ignore self weight.

Compute the maximum load that can be resisted by the beam. Assume the reinforcing bars used are Grade 60 and assume that the strength of concrete in compression does not limit the flexural strength of the section.

Given:

- \( f_y = 60\text{ksi} \) yield stress of reinforcement
- \( A_s = 3\text{in}^2 \) area of reinforcing steel
- \( L = 24\text{ft} \) length of beam
- \( d = 24\text{in} \) depth to reinforcement
- \( jd = d - 0.1d \) internal lever arm to reinforcement

\( jd = 21.6\text{-in} \)
Solution:

Compute maximum moment (assuming that the strength of the concrete in compression does not limit the flexural strength of the section).

\[ M = Tjd \quad \text{moment} \]

\[ T = f_y A_s \quad \text{nominal tensile capacity of reinforcement} \]

\[ M_u := f_y A_s j_d \quad \text{because concrete does not control the failure mechanism, we assume the reinforcement reaches its yield stress} \]

\[ M_u = 324 \text{-kip-ft} \]

Compute the maximum load

\[ M = \frac{P L}{4} \quad \text{moment caused by point load} \]

\[ P := \frac{M_u 4}{L} \]

\[ P = 54 \text{-kip} \]