Exam 2

Short answer (10 points)

1. Sketch 2 different frames below that would be considered sway-frames and why they are treated as such.

2. In a sway frame, what assumption do we make that allows us to treat the displacements on each side of the frame equal?

   \[ \text{no axial shortening in the beam} \]

3. For frames with different moments of inertia and lengths, we often substitute a stiffness factor \( k \) for what variables in the slope deflection equation? (i.e. what does \( k \) equal?)

   \[ k = \frac{EI}{L} \]

4. Why do we have a greater appreciation for the Empire State Building?

   \[ \text{It was likely designed using slope deflection methods.} \]

5. What time period was the slope deflection method used to analyze statically indeterminate structures?

   \[ 1915 - 1930 \]

Problem 1 (30 pts)

a) Setup the system of equations (in terms of \( E \), \( I \), and \( L \)) you would use to solve for the moments in the beam below. The support at B settles 0.15 ft.

b) If the moment at A \( (M_{AB}) \) is -2740 k-ft, calculate the rotation at B \( (\theta_B) \).

\[ E = 29000 \ \text{ksi} \]
\[ I = 8000 \ \text{in}^4 \]

\[ \begin{array}{c}
\text{0.24 k/ft} \\
\text{A} \\
\hline
\text{20 ft} \\
\text{B} \\
\hline
\text{0.3 k/ft} \\
\text{C} \\
\end{array} \]
Problem 2 (30 points)
Solve for the end moments in the frame below.

\[ E = 29000 \text{ ksi} \]
\[ I_{AB} = 700 \text{ in}^4 \]
\[ I_{CD} = 1100 \text{ in}^4 \]

![Frame diagram](image)

Problem 3 (30 points)
Setup the system of equations you would use to solve for the moments at the ends of each member.

![Frame diagram](image)
Exam 2 Solution:

(1) Fixed end moments:

\[ F_{EM_{AB}} = \frac{wL^2}{12} - \frac{24(20)^2}{12} = -8 \text{ k-ft} \]

\[ F_{EM_{BA}} = +8 \text{ k-ft} \]

\[ F_{EM_{BC}} = \frac{wL^2}{8} = \frac{0.3(30)^2}{8} = -33.8 \text{ k-ft} \]

\[ F_{EM_{CB}} = 0 \]

\[ \psi_{AB} = \frac{\Delta}{L} = \frac{0.15 \mu}{20 \text{ ft}} = 0.0075 \]

\[ \psi_{BC} = \frac{0.15 \mu}{30 \text{ ft}} = 0.005 \]

**Deflected shape:**

- Counterclockwise chord rotation,
  \[ \psi_{BC} = \Theta 0.005 \]
- Clockwise chord rotation,
  \[ \psi_{AB} = \psi_{BA} = \Theta 0.0075 \]
Slope deflection equations:

\[ M_{AB} = \frac{2EI}{L} \left( \theta_A + \theta_B - 3\psi_{AB} \right) - 8 \]

\[ M_{AB} = \frac{2EI}{L} \left( \theta_B - 3\psi_{AB} \right) - 8 \quad (1) \]

\[ M_{BA} = \frac{2EI}{L} \left( \theta_A + 2\theta_B - 3\psi_{BA} \right) + 8 \]

\[ M_{BA} = \frac{2EI}{L} \left( 2\theta_B - 3\psi_{BA} \right) + 8 \quad (2) \]

\[ M_{BC} = \frac{3EI}{L} \left( \theta_B - \psi_{BC} \right) - 33.8 \quad (3) \]

3 Equations, 4 unknowns:

\[ M_{BA} + M_{BC} = 0 \quad (4) \]

b.) \[ E = 29,000 \text{ ksi}, \quad I = 8000 \text{ in}^4 \]

\[ M_{AB} = -2740 \text{ k-ft} \]

\[ M_{AB} = -2740 \text{ k-ft} = \frac{2(29000 \text{ ksi})(8000 \text{ in}^4)(0.00075)}{20 \text{ ft}^2} \left( \theta_B - 3\psi_{AB} \right) - 8 \text{ ft} \]

\[ -2740 \text{ k-ft} = 161,111 \text{ k-ft} \left( \theta_B - 0.023 \right) - 8 \text{ k-ft} \]

\[ \theta_B = 0.006 \]
2. Fifth end moments:

\[ FEM_{BD} = \frac{PL}{3} = \frac{3k(96)}{3} = -9k\cdot ft = -108k\cdot in \]

Slip deflection equations:

\[ M_{BA} = \frac{3EI}{L} (\theta_B - \theta) \]

\[ M_{BA} = \frac{3(29000)(700)}{18(12)} \theta_B \]

\[ M_{BA} = 281,944 \theta_B \] (1)

\[ M_{BC} = \frac{2EI}{L} (2\theta_B + \theta_C - 3\theta) \]

\[ M_{BC} = \frac{2(29000)(700)}{12(12)} (2\theta_B) \]

\[ M_{BC} = 563,889 \theta_B \] (2)

\[ M_{CB} = \frac{2EI}{L} (\theta_B + 2\theta_C - 3\theta) \]

\[ M_{CB} = 281,944 \theta_B \] (3)

\[ M_{BD} = \frac{3EI}{L} (\theta_B - 3\theta) - 108k\cdot in \]

\[ M_{BD} = 886,111 \theta_B - 108 \] (4)
4 equations, 5 unknowns

\[ MBA + M_{BC} + M_{BD} = 0 \quad (5) \]

Substitute equations (1), (2), (4) into (5)

\[ 281,944 \theta_B + 563,889 \theta_B + 886,111 \theta_B - 108 = 0 \]

\[ \theta_B = 0.0000624 \]

\[ \begin{align*}
M_{BA} &= 17.6 \text{ k-in} \\
M_{BC} &= 35.2 \text{ k-in} \\
M_{CB} &= 17.6 \text{ k-in} \\
M_{BD} &= -52.7 \text{ k-in}
\end{align*} \]
Torsional moments:

\[ FEM_{AB} = \frac{Wl^2}{20} = \frac{1.2 \frac{L}{L} (30)^2}{20} = 54 \text{ k} \cdot \text{ft} \]

\[ FEM_{BA} = \frac{Wl^2}{30} = \frac{1.2 \frac{L}{L} (30)^2}{30} = 36 \text{ k} \cdot \text{ft} \]

\[ M_{AB} = \frac{2EI}{L} \left( 2\theta_A + \theta_B - 3\psi \right) - 54 \]

\[ M_{AB} = \frac{2EI}{L} (\theta_B - 3\psi) - 54 \quad (1) \]

\[ M_{BA} = \frac{2EI}{L} \left( 2\theta_B - 3\psi \right) + 36 \quad (2) \]

\[ M_{BC} = \frac{2EI}{L} \left( 2\theta_B + \theta_C \right) \quad (3) \]

\[ M_{CB} = \frac{2EI}{L} (\theta_B + 2\theta_C) \quad (4) \]

\[ M_{CD} = \frac{3EI}{L} (\theta_C - \psi) \quad (5) \]

5 equations, 8 unknowns.

\[ M_{BA} + M_{BC} = 0 \quad (6) \]

\[ M_{AB} + M_{CD} = 0 \quad (7) \]
Consider free-body of each column.

\[ \sum M_B = MBA + MAB + 18(20) - H_A(30) = 0 \]

\[ H_A = \frac{MBA + MAB + 360}{30} \]

\[ \sum M_C = MCD - H_D(30) = 0 \]

\[ H_D = \frac{MCD}{30} \]

\[ \sum F_x = -\left(\frac{MBA + MAB + 360}{30}\right) - \left(\frac{MCD}{30}\right) + 18 = 0 \]