Exam 2

Allowed: 1 page review sheet, ACI 318-08 Code

Short Answer (25 points):
1. Label the curvature on the strain diagram below and an expression to calculate it in terms of the variables shown.

\[ \phi = \frac{\epsilon_c + \epsilon_s}{d} \]

2. Locate and label the stages of behavior on the moment-curvature diagram below:
3. The ACI code assumes the Whitney stress block for the design of reinforced concrete flexural members. Please sketch on the diagram below the shape of this stress block and label the variables a, c, and an expression for β₁.

\[ C = \frac{a}{B_1} \]

4. The reinforcement ratio for most flexural members is approximately 1.0 %

5. The simplified method of constructing the limiting moment and curvature values include the following 2 assumptions:
   - mean compressive stress in concrete = 0.7 \( f'_c \)
   - location of the compressive force resultant = 0.4 \( c \)

6. The value j for reinforced concrete flexural members can be approximated as 0.9.

7. Estimating the weight of reinforced members can be done several ways. Please list one method a designer would use to estimate the weight.

\[ h \approx \frac{L}{12} \quad b \approx \frac{h}{2} \]

8. ACI 10.5.1 requires a minimum amount of reinforcement for flexural members. Why?

To avoid brittle failure

9. Concrete reaches its maximum stress at a strain of approximately 0.002.

10. An approximate value for the modulus of elasticity of concrete is 3600 ksi.
Problem 1 (25 points)
Consider the two cross sections below. Assume that you designed a beam with 5 - #7 bars, but an error occurred in the field, and it was constructed with 5 - #11 bars. Because this was an obvious construction error and the error was not identified until construction was nearly complete, the building official will allow you to leave the as-built beam in place rather than removing it, if you can demonstrate the flexural capacity of the as-built beam in place is at least 1.4 times the flexural capacity of the as-designed beam. (Note, this is not a provision in the building code, but simply a problem statement.)

![Cross sections of beams](image)

Assume the compressive strength of the concrete is 4000 psi and the yield stress of the reinforcement is 60 ksi.

a.) Determine \( M_n \) for the as-designed beam.
b.) Determine \( M_n \) for the as-built beam.
c.) Calculate the tensile strain, \( \varepsilon_t \), in the reinforcement for both sections.
d.) Please use ACI Figure R9.3.2 below to calculate the \( \phi \)-factor for both beams.
e.) Will the building official allow you to leave the as-built beam in place?

![\( \phi \) values](image)
Problem 2 (25 points):
Moments for the cross section are provided in the moment-curvature diagram below.

a.) Calculate the curvatures corresponding to cracked, yielded, and limiting conditions of the cross section. You may use the simplified procedure for the limiting curvature value.
b.) Based on the shape of the moment curvature plot, is it likely the concrete or reinforcement failure controls the cross section? Why?

Problem 3 (25 points):

Design the shear reinforcement for the beam. Please locate and label your reinforcement on the beam sketch above. Use # 3 stirrups.
1. a) \[ M_m = A_s f_y \left( d - \frac{a}{2} \right) \]

\[ A_s = 5(0.60 \text{ in}^2) - 3.0 \text{ in}^2 \]

\[ f_y = 60 \text{ ksi} \]

\[ d = 18.5 \text{ in} \]

\[ a = \frac{A_s f_y}{0.85 f_c b} = \frac{3.0 \text{ in}^2(60 \text{ ksi})}{0.85(4 \text{ ksi})(14 \text{ in})} \]

\[ a = 3.8 \text{ in} \]

\[ M_m = 3.0 \text{ in}^2(60 \text{ ksi})(18.5 \text{ in} - \frac{3.8 \text{ in}}{2}) \]

\[ M_m = 2988 \text{ kN \cdot m} = 249 \text{ kN \cdot ft} \]

1. b) \[ M_m = A_s f_y \left( d - \frac{a}{2} \right) \]

\[ A_s = 5(0.56 \text{ in}^2) = 7.8 \text{ in}^2 \]

\[ f_y = 60 \text{ ksi} \]

\[ a = \frac{7.8 \text{ in}^2(60 \text{ ksi})}{0.85(4 \text{ ksi})(14 \text{ in})} = 9.8 \text{ in} \]

\[ M_m = 7.8 \text{ in}^2(60 \text{ ksi})(18.5 - \frac{9.8 \text{ in}}{2}) \]

\[ M_m = 6358 \text{ kN \cdot m} = 530 \text{ kN \cdot ft} \]
c.) \( a = \frac{E_c}{E_t} = \frac{3.8}{0.85} = 4.47 \)

\( c = \frac{a}{2} = \frac{4.47}{2} = 2.24 \text{ in} \)

\( B_1 = 0.85 \text{ for } f_c = 4000 \text{ psi} \)

\( \frac{0.003}{4.47} = \frac{E_t}{18.5-4.47} \)

\( E_t = 0.009 > 0.005 \)

\( \Rightarrow \text{tension controlled} \)

\( d.) \ \phi = 0.9 \text{ as-designed} \)

\( \phi = 0.65 \text{ as-built} \)

\( e.) \text{ as-designed} \)

\( \phi M_m = 0.9(249 \text{ k-ft}) = 224 \text{ k-ft} \)

\( 1.4(224.1) = 313 \text{ k-ft} < 344 \text{ k-ft} \)

\( \text{Beam OK} \)
Given:

- Width = 18 in
- Depth = 14 in
- Section = 4.0 in²

Shear:

\[ M_{sv} = \frac{ft \cdot g}{y} = \frac{7.5 \sqrt{4000} \left( \frac{14(18)^3}{12} \right)}{20.5} \left( \frac{1}{1000/16} \right) \left( \frac{16}{12 \text{ in}} \right) \]

\[ M_{sv} = 26.2 \text{ kips} \]

Bending Moment:

\[ M_y = Asfyd \left( 1 - \frac{k_d}{3} \right) = 4.0(60 \text{ kips})(18 \text{ in})(1 - \frac{0.016}{3}) \]

Shear Force:

\[ R = \sqrt{0.016^2(8.1)^2 + 2(0.016)(8.1)} = 8.1(0.016) \]

\[ R = 0.40 \text{ kips} \]

Bending Moment:

\[ M_y = 4.0(60 \times 18 \text{ in})(1 - \frac{0.4}{3}) = 312 \text{ kips} \]
\[ M_n = A_s f_y (1 - 0.4 k_u) d \]

\[ k_u = \frac{A_s f_y}{0.7 f_c b d} = \frac{4.0(60)}{0.7(4)(14)(18)} = 0.34 \]

\[ M_n = 4.0(60)(1 - 0.4(0.34))(18) = 311 \text{ k-ft} \]
(2.)

a.) \[ \Phi_{cr} = \frac{M_{cr}}{E_c I_g} \]

\[ I_g = \frac{bh^3}{12} = \frac{14(20.5)^3}{12} = 10,050 \text{ in}^4 \]

\[ E_c = 3600 \text{ ksi} \]

\[ \Phi_{cr} = \frac{20.2 \text{ kips} (12 \text{ in})}{3600 \text{ ksi} (10,050 \text{ in}^4)} \]

\[ \Phi_{cr} = 0.0000087 \]

\[ \Phi y = \frac{M y}{E c I c r} = \frac{E c (\frac{1}{2} b k d^2 (1 - \frac{r}{h}) k d}{M y} \]

\[ k = \sqrt{p_n^2 n^2 + 2 p_n - p_n} \]

\[ p = \frac{A_s}{b d} = \frac{4(1.0 \text{ in}^2)}{14 \text{ in}(18 \text{ in})} = 0.016 \]

\[ n = \frac{E_s}{E_c} = \frac{29000}{3600} = 8.1 \]

\[ k = \sqrt{0.016^2 (8.1)^2 + 2 (0.016)(8.1) - 0.016(8.1)} \]

\[ k = 0.40 \]
\[ \phi_y = \frac{312(12)}{3600(\frac{1}{2})(14)(0.4)(18)} \left( 1 - \frac{0.4}{3} \right) \frac{0.4}{18} \text{ in} \]

\[ \phi_y = 0.00018 \]

\[ \phi_u = \frac{E_e u}{k_n \cdot d} \]

\[ k_n = 0.7 f_c b d \]

\[ k_n = 0.34 \]

\[ \phi_u = \frac{0.003}{0.34(18)} \]

\[ \phi_u = 0.00049 \]

b.) Likely a compression failure. Steel failure would result in higher moment due to strain hardening.

[Diagram with annotations for steel rupture and concrete crushing]
$$P_u = 1.2 (60k) + 1.6 (40k)$$

$$P_u = 136k \rightarrow V_u = \frac{1}{2} (136) = 68k$$

$$\Phi V_c = \left[2 \sqrt{4000 \cdot 16 (30in)} \right] 0.75$$

$$\Phi V_c = 45.5k < V_u \rightarrow \text{shear verify reqd.}$$

$$V_c = 61k$$

$$\frac{V_u}{\Phi} = V_c + V_s$$

$$\frac{68k}{0.75} = 61k + V_s$$

$$V_s = 30k = \frac{A_v f_y d}{s} = \frac{0.22 in^2 (60 ksi) (30 in)}{s}$$

$$s = 13.2 in$$

$$s_{min} = \frac{d}{2} = \frac{30 in}{2} = 15 in$$

$$\rightarrow \text{use 13 in sturup spacing}$$