Reinforced concrete structures that are cast in place are typically monolithic. Casting is accomplished through a series of individual lifts but the entire structure is one continuous element. A slab cast on a beam (Fig. 18-1) becomes the top flange of the beam. The beam is said to have a T-section\(^1\).

Section 18 will focus on the proportioning of beams with T-sections. We shall be concerned with the determinations of the moment and unit curvature capacities of the section. Before we tackle the subject, there are four issues that we should note.

1. The flexural response of a T-section subjected to negative bending moment (top flange in tension) is essentially the same as that of a rectangular section except for the moment and curvature at cracking. The moments and curvatures at yield and at the limiting condition may be determined as described in Section 14. It is, however, important to note that the flange can accommodate tensile reinforcement conveniently.

2. In T-sections subjected to positive bending moment (top flange in compression), if the depth to the neutral axis, at yield and at the limit, calculated using the expressions for a rectangular section is not more than the flange thickness, \(t_f\), flexural response at yield and at the limit can be determined using the expressions for rectangular sections.

3. If the designer elects to ignore the effect of the top flange in determining the required amount of tensile reinforcement, it is highly unlikely that any existing code authority will fault the designer.

4. In cases where the beam spacing is large, the designer needs to define the effective width of the top flange. A pragmatic rule is to assume that the flange on a side is equal to the depth of the beam below the slab. The thickness of the slab also influences the flange width. Building codes tend to limit the flange, on each side, to less than 3 to 8 times the slab thickness. The range of opinion reflects the variability of analysis and judgment. For light to moderate tensile reinforcement, decisions about the flange width make very little difference on proportioning of the section as implied by the freedom given the designer for ignoring the flange. Of course, the flange width may not extend more than half the clear distance to the next parallel beam. In all cases, the choice of the flange-width dimension must conform to the applicable code.

\(^1\) If the beam is located at the edge of the slab, it is said to have an L-section.
A T-Section Subjected to Positive Moment

To determine the moment and curvature capacity, we assume that the strain distribution is linear over the section, and we project this assumption to the overhanging portions of the flange.

If the neutral axis falls within the flange \((k_u d < t_f)\), the expressions derived for rectangular sections apply except that the width of the area of concrete in compression is not \(b\) but \(b_f\). Therefore, we redefine \(\rho\):

\[
\rho_f = \frac{A_s}{b_f d}
\]

Eq. 18-1

We determine the relative depth to the neutral axis from equilibrium of normal forces on the section as we did for a rectangular section:

\[
k_u = \frac{\rho_f \cdot f'_c}{k_2 \cdot 0.85 \cdot f'_c}
\]

Eq. 18-2

With this definition of \(k_u\), the expressions derived in Section 14 for the relative internal moment arm \((j)\) and moment capacity of rectangular beams remain applicable.

The designer should realize that \(\rho_f\) is a fraction of \(\rho\) and it is not an indicator of whether the reinforcement will fit within the web.

If the calculated ratio \(k_u\) exceeds the ratio \(t_f / d\), to be consistent with assumptions made, then, we divide the area in compression into two regions (Fig. 18-3). The resultant of stresses in region 1 (within the web) is determined as the product of the mean compressive stress in the concrete, \(k_1 0.85 f'_c\), the web thickness, \(b_w\), and the depth to the neutral axis \(k_u d\):

\[
C_c = k_1 \cdot 0.85 f'_c \cdot b \cdot k_u d
\]

Eq. 18-3
If we consider the assumptions made in determining the flange width and the variation of stress to be accurate, the resultant of stresses in region 2 requires work because it depends on the ratio of flange thickness to neutral axis depth. Because the error introduced is small, and because the width $b_f$ is not exact, we simply assume the force on region 2 to be:

$$C_f = 0.85f'_c \cdot (b_f - b) \cdot t_f$$  \hspace{1cm} (18-4)

In essence, our assumption implies that the two 3D objects shown in Figure 18-4 have the same volume. They do not. The “wings” of the lower object are too large. The centroids of these objects do not coincide either. We ignore these discrepancies to write an expression describing equilibrium of axial forces similar to the expression we wrote for beams with compression reinforcement:

$$C_c + C_f = T$$  \hspace{1cm} Eq. 18-5

or

$$k_i \cdot 0.85f'_c \cdot b \cdot k_u d + 0.85f'_c \cdot (b_f - b) \cdot t_f = A_s f_y$$  \hspace{1cm} Eq. 18-6

Solving for $k_u$: and simplifying

$$k_u = \frac{\rho \cdot f_y}{k_i \cdot 0.85f'_c} - \frac{1}{k_i d} \left( \frac{b_f}{b} - 1 \right)$$  \hspace{1cm} Eq. 18-7
The term \( \frac{1}{k_1} \frac{t_f}{d} \left( \frac{b_f}{b} - 1 \right) \) represents a shift of the neutral axis caused by the effect of the slab. If this shift is not large enough to make the product \( k_1 k_a d \) larger than \( t_f \), then we apply again the expressions derived for rectangular beams (as modified above).

For \( \rho < 2\% \)

\( f_y = 60\text{ksi} \)

\( f'_c > 4\text{ksi} \)

the term \( \frac{\rho \cdot f_y}{k_1 \cdot 0.85f'_c} \) is smaller than \( \sim 2/5 \). We replace this value \( (2/5) \) in the expression we derived for \( k_u \) and conclude that, within the defined ranges, if \( t_f > \frac{2}{5} k_1 \frac{b}{b_f} d \), then the depth of the equivalent compression block \( (k_1 k_a d) \) is smaller than the thickness of the flange \( (t_f) \) and we can treat the section as if it was rectangular.

The function \( \frac{2}{5} \cdot k_1 \frac{b}{b_f} \) is plotted vs. \( \frac{b}{b_f} \) for \( \frac{b}{b_f} < \frac{1}{3} \) and \( k_1=0.85 \) in Figure 18-5. From this plot we conclude that, within the ranges we have defined, \( T \) beams with \( t_f/d \) larger than \( 1/8 \) can be treated as rectangular beams (using a revised reinforcement ratio: \( \rho = \frac{A_s}{b_f \cdot d} \)). We need to examine other cases more closely. If the depth of the equivalent stress block falls outside the flange, moment capacity is computed as:

\[
M_n = C_c \cdot \left( 1 - \frac{1}{2} k_1 \cdot k_u \right) \cdot d + C_f \cdot \left( d - \frac{1}{2} t_f \right)
\]

Eq. 18-8

The moment, \( M_n \), can be estimated closely as \( M_n = A_s \cdot f_y \cdot 0.9d \) unless the section is “very heavily” reinforced.

![Figure 18-5. Domain in which the Depth of the “Compression Block” is Smaller than Flange Thickness](image-url)
Example

Compute the moment capacity of the section shown assuming $f'_c = 5000$ psi

Solution

If we assume we can treat the section as if it was rectangular we compute:

\[
\rho = \frac{A_s}{b_f \cdot d} = \frac{3\text{in}^2}{48\text{in} \cdot 2\text{lin}} = 0.3\% \text{ and }
\]

\[
k_u = \frac{\rho \cdot f_y}{k_1 \cdot 0.85f'_c} = \frac{0.003 \cdot 60\text{ksi}}{0.8 \cdot 0.85 \cdot 5\text{ksi}} = 0.053 < 0.6 \text{ (and, therefore, } \rho_s = \rho_y)\]

The depth of the equivalent compression block would be $k_1 k_u d = (0.8 \times 0.053) \times 21\text{in.} = 0.042 \times 21\text{in.} = 0.9 \text{ in.}$ This is (much) smaller than the thickness of the flange $t_f = 6 \text{ in.}$ We should have expected this result because $t_f/d = 0.29 > 1/8$.

The internal lever arm is:

\[
j = 1 - k_1 k_u / 2 = 1 - 0.02 = 0.98
\]

The nominal moment capacity is:

\[
M_n = A_s f_y j d = 3\text{in}^2 \cdot 60\text{ksi} \cdot 0.98 \cdot 21\text{lin.} = 3700\text{kip} \cdot \text{in.}
\]

Observe that this result is again close (the error is 8%) to $A_s f_y 0.9d = 3\text{in}^2 60\text{ksi} 0.9 21\text{in}= 3400 \text{ kip in.}$

Exercise

Repeat the exercise above for $f'_c = 3000$ psi and a flange thickness of 4 in.

Essentials: If $t_f > d/8$, we can usually treat beams with T-shaped sections as beams with rectangular sections with a reinforcement ratio $\rho = \frac{A_s}{b_f \cdot d}$.  

Reinforced Concrete in Thirty Lectures 18-5