Section 27. Control of Flexural Cracks

Widths of flexural cracks at service loads need to be considered in design (1) if the structural element is visible and (2) if it is exposed to a corrosive environment. One may get the impression from engineering literature that flexural-cracking consideration in design requires calculation of crack widths to a precision of 0.001 in. We shall make no such pretense. Our goal is to provide a simple understanding of the dominant factors affecting flexural-crack widths.

Tensile reinforcement cut from Grade 60 steel may sustain strains as high as 0.0015 at service load. The limiting tensile strain of concrete is unlikely to exceed 0.0002. Cracking of the concrete on the tension side of elements subjected to bending cannot be avoided under normal circumstances but crack widths may be controlled by proper proportioning.

To try and understand any poorly understood phenomenon, it is wise to look at its manifestations and see if results can be organized in terms of variables that affect them. Flexural cracking of reinforced concrete elements is certainly one of those poorly understood phenomena.

Figure 27-1

Figure 27-2
Figure 27-1 shows the cross-sectional properties of a test girder. Its elevation is shown in Fig. 27-2 including the cracks in the central span between the applied concentrated loads. Thinking of the distribution of flexural strain over the depth of the girder, one would expect the crack width to increase at a constant rate from the neutral axis to the tension flange. Because of the stiffening effect of the reinforcement,

![Figure 27-3](image)

the crack increases in width with distance from the neutral axis toward the tension flange but is pinched at the level of the tensile reinforcement and starts expanding again as it approaches the extreme fiber in tension (Fig. 27-3). In this section, the crack width we refer to will be that at the level of the reinforcement.

The part of the span between the applied loads of the test girder in Fig. 27-1 was subjected to essentially constant moment at loads approaching and exceeding the service load. The bending moment demand from point A to point C varied very little suggesting that the reinforcement stress (and, therefore, strain) remained essentially constant. The crack widths measured at reinforcement level at both faces of the beam between the applied loads are shown in Fig. 27-4. The widths shown were measured at reinforcement stresses of 31 and 36 ksi. Unquestionably, the most striking aspect of the crack-width distribution is that the ratio of the maximum to the minimum measured crack width approached an order of magnitude in both cases.

Why should the scatter be so large? If we think of the cracking phenomenon as resulting from the transfer of stress from the reinforcement to the concrete surrounding it, the scatter appears plausible. The stress transfer must occur through a process akin to friction and cracking must occur when sufficient stress has built up in the concrete to reach its tensile strength. We have no reason to assume that either phenomenon is uniform along the beam. Crack spacing is bound to be random. If we think of the crack width as the accumulation of the strain over a distance equal to the crack spacing, it follows that it is unreasonable to accept all cracks to have the same width even if the strain in the reinforcement is reasonably uniform.
Figure 27-4
Figure 27-5

Figure 27-5 shows the variation of crack width (at reinforcement level) with nominal reinforcement stress. The data indicate clearly that there was a direct relationship between crack width and stress in the tensile reinforcement. We also note that for the test girder considered, the maximum crack width was approximately twice the mean crack width. It is also of interest to observe that the “reference crack widths,” defined as the mean width plus two standard deviations, were quite comparable to the maximum crack widths up to a reinforcement stress of 40 ksi.

To control cracking, we need to identify the dominant factors that drive the width of the crack. One of the factors is the tensile stress in the steel as is evident from Fig. 27-5. The width of the crack must be proportional to the extension of the reinforcement between two adjacent cracks. If we can identify the factors that influence crack spacing, we shall be able to develop a procedure to control crack widths.

To establish the factors influencing crack spacing, we consider a segment of a girder between two flexural cracks (Fig. 27-6) at A and C and ask whether another crack will form at B, a point equidistant to the existing cracks.

The tensile stress in the concrete at A, at one face of the existing crack, is zero. We reason that if the tensile stress at a vertical section crossing B can reach the effective tensile strength of the concrete, a crack may form at B. The tensile force at that section may be defined approximately as

\[ T_{FB} = A_c K_1 f'c \]  

Eq. 27-1

\[ T_{FB} = \text{Tensile force required to cause a flexural crack at B} \]
\[ A_e = \text{Area on vertical section at B over which tensile stress exists.} \]

\[ K_1 = \text{A factor that averages the magnitude and distribution of the tensile stress acting on section at B over area } A_e. \]

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**Figure 27-6**

The reinforcing-bar stress at A will be transferred to the concrete surrounding the bar along the length \( \lambda_{AB} \). We can express the total force transferred as

\[ T_{tr} = \pi d_b K_2 f'_c \lambda_{AB} \quad \text{Eq. 27-2} \]

- \( T_{tr} \): Tensile force transmitted from the reinforcement to the concrete per unit length
- \( d_b \): diameter of bar(s)
- \( \lambda_{AB} \): Distance from A to B
- \( K_2 \): A factor that averages the magnitude, along length \( \lambda_{AB} \), of the force transferred.

Ideally, a flexural crack will form at B if

\[ \lambda_{AB} = \frac{A_e K_1 f'_c}{\pi d_b K_2 f'_c} \quad \text{Eq. 27-3} \]

The expression above does not justify our calculating the magnitude of \( \lambda_{AB} \), but it does help organize the critical factors for determining the crack spacing. We go further. We assume that \( K_1 \) and \( K_2 \) are equal. That assumption helps us reduce the expression to

\[ \lambda_{AB} = \frac{A_e}{\pi d_b} \quad \text{Eq. 27-4} \]
We do not know the area $A_e$ but we know that if we divide the area by $d_b$, we shall obtain a length. What could that length be? We refer to an experimental study of cracking by Broms\(^1\) to infer it must be a function of the distance $c_b$ from the extreme fiber in tension to the center of the reinforcing bar, provided the lateral spacing of the bars is approximately $2c_b$.

We assume conservatively that the reinforcement strain is constant along the reinforcing bar. The crack width is then proportional to

$$c_b \frac{f_s}{E_s}$$

$c_b$ : concrete cover

$f_s$ : stress in reinforcement at cracked section

$E_s$: Young's modulus for reinforcement

The expression above identifies the dominant factors that control crack width and suggests the following design choices.

If the design criterion relates to visual appearance of the reinforced concrete member subjected to tensile stresses, the preferred action is to reduce the tensile stress (by adding reinforcement) to a minimum and/or to use the minimum cover required for other requirements reasons (such as fire protection) in the applicable building code.

If the design criterion is corrosion resistance, the reinforcement stress should be reduced but the cover, if at all possible, should be increased.

What do we conclude from this foray into estimating flexural crack widths?

- Determining crack width starting from first principles may be achieved if one ignores the evidence.
- The maximum crack width is likely to be less than twice the mean crack width.
- The mean crack width may be estimated as a product of the mean crack spacing and the steel strain.
- The mean crack spacing may be estimated as a multiple of the concrete cover. Twice the concrete cover is a good “starting estimate.”
- At service-load level, the crack width may be considered to vary linearly from zero at the neutral axis to a maximum at the extreme fiber in tension.