Problem 1-1

If 3 kW is conducted through a section of insulating material 0.6 m² in cross section and 2.5cm thick and the thermal conductivity may be taken as 0.2 W/m°C, compute the temperature difference across the material.

Sketch:

\[ \begin{array}{c}
0.6 \text{ m}^2 \\
0.025 \text{ m}
\end{array} \]

\[ K = 0.2 \ \text{ W/m°C} \]

\[ q = 3 \ \text{kW} = 3000 \ \text{W} \]

Assumptions: System is in steady state, Conductivity is independent of temperature.

Equations: Conductivity equation \([1-1]\) \[ q = -KA \frac{dT}{dx} \]

Solution:

\[ q^2 = -KA \frac{dT}{dx} \]

\[ \frac{dT}{dx} \]

\[ \frac{dT}{dx} = -KA \frac{T_1 - T_2}{0.025 \text{ m}} \]

\[ T_1 - T_2 = \frac{\Delta x}{-KA} \]

\[ \Delta T = \frac{\Delta x}{-KA} \left( 0.025 \text{ m} \times 3000 \text{ W} \right) \]

\[ = -625 \ ^\circ \text{C} \]

* In this problem, the direction of heat flow is not specified, so the sign of \( \Delta T \) is meaningless.

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Problem 1-5

A certain superinsulation material having a thermal conductivity of \( 2 \times 10^{-4} \ \text{ W/m°C} \) is used to insulate a tank of liquid nitrogen that is maintained at \( -196^\circ \text{C} \). 199 kJ is required to vaporize each kilogram of nitrogen at this temperature. Assuming that the tank is a sphere having an inner diameter of 0.52 m, estimate the amount of nitrogen vaporized per day for an insulation thickness of 2.5 cm and an ambient temperature of 21°C. Assume that the outer temperature of the insulation is 21°C.

Sketch:

\[ \begin{array}{c}
T_i \\
T_o
\end{array} \]

\[ K = 2 \times 10^{-4} \ \text{ W/m°C} \]

\[ K_i = -196 ^\circ \text{C} \]

\[ T_o = 21 ^\circ \text{C} \]

\[ H_{fg} = 199 \ \text{ kJ/kg (latent heat)} \]

Assumptions: System is in steady state, Conduction only, no convection effects, Spherical Symmetry in \( \phi, \theta \). Temperature is only a function of \( r \)
Problem 1-5 (continued)

Equations: \[ q = -KA \frac{dT}{dr} \] Conductivity equation in spherical coordinates

\[ \frac{1}{r} \frac{d^2(rT)}{dr^2} + \frac{1}{r^2 \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{dT}{d\theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{d^2 T}{dr^2} + \frac{q}{k} = \frac{1}{r^2} \frac{dT}{dr} \]

\[ \frac{1}{r} \frac{d^2(rT)}{dr^2} = 0 \]

Solution:

Differential equation,

\[ \frac{1}{r} \frac{d^2(rT)}{dr^2} = 0 \]

\[ rT = C_1 r + C_2 \]

\[ T = \frac{C_1 r + C_2}{r} \]

plug into \[ q = -KA \frac{dT}{dr} \]

\[ \frac{dT}{dr} = -\frac{C_2}{r^2} \]

So we must find \( C_2 \). Use boundary conditions

\[ T_o = C_1 + \frac{C_2}{r_o} \]

\[ T_i = C_1 + \frac{C_2}{r_i} \]

\[ T_o - T_i = \frac{C_2}{r_o} - \frac{C_2}{r_i} \]

\[ C_2 = \frac{T_o - T_i}{\left( \frac{1}{r_o} - \frac{1}{r_i} \right)} \]

\[ \frac{dT}{dr} = -\frac{1}{r^2} \left( \frac{T_o - T_i}{\left( \frac{1}{r_o} - \frac{1}{r_i} \right)} \right) \]

\[ A = 4\pi r^2 \] Surface area of a sphere

\[ q = -KA \frac{dT}{dr} = -K \left( 4\pi r^2 \right) \frac{-1}{r^2} \left( \frac{T_o - T_i}{\left( \frac{1}{r_o} - \frac{1}{r_i} \right)} \right) \]

\[ q = -K \left( 4\pi \right) \frac{T_o - T_i}{\left( \frac{1}{r_o} - \frac{1}{r_i} \right)} = -2 \times 10^{-4} \text{ W/m}^2 \]

\[ \text{Mass vaporized per day} = \text{1 day (24 hr)/day} \times \frac{3600 \text{ s/hr}}{(1.62 \text{ W}) \left( \frac{\text{w}}{\text{m}^2} \right)} \]

\[ m = \frac{K_j}{199 K_5} \times 1.62 \text{ W} \times \left( \frac{3600 \text{ s}}{\text{hr}} \right) \times \left( \frac{\text{kg}}{10^5 \text{ J}} \right) = 8.1 \times 10^{-6} \text{ kg/d} \]

\[ \text{Kg/day} = 8.1 \times 10^{-6} \text{ kg/(hr)} \left( 3600 \text{ s/hr} \right) \]

\[ \text{Kg/day} = 0.7 \text{ Kg/day} \]
Problem 1.9

A certain insulation has a thermal conductivity of $10 \text{ W/m}^2\cdot\text{K}$. What thickness is necessary to effect a temperature drop of 500°C for a heat flow of $400 \text{ W/m}^2$?

Sketch:

Assumptions: Steady state, no generation, Cartesian geometry, flat plate, 1-dimensional model.

Equations: Conductivity equation $q = -kA \frac{dT}{dx}$

Solution:

$$q = -k \frac{dT}{dx} \Rightarrow \Delta x = \frac{-q}{k} \frac{dT}{dx}$$

$$\Delta x = \frac{-400 \text{ W/m}^2 \cdot \text{K}}{10 \text{ W/m}^2 \cdot \text{K}} \frac{500\degree \text{C}}{400 \text{ W/m}^2}$$

$$\Delta x = 12.5 \text{ m}$$

Problem 1.15

Water flows at a rate of 0.5 kg/s in a 2.5 cm diameter tube having a length of 3 m. A constant heat flux is imposed at the tube wall so that the tube wall temperature is 40°C higher than the water temperature. Calculate the heat transfer and estimate the temperature rise in the water. The water is pressurized so that boiling cannot occur.

Sketch:

Assumptions: Steady state, No generation, Hugoniot.

Equations: Area of cylinder $A = \pi \phi h$

Conduction equation $q = -kA \frac{dT}{dx}$

Convection equation $q = hA(T_w - T_o)$

$$h = 3500 \text{ Table 1 P11}$$

$$h = 4186 \frac{W}{\text{m}^2\cdot\text{K}}$$

$$\Delta T = \left(1 - \frac{8}{10}\right) \frac{1 - 8\frac{^2 \text{C}}{^2 \text{K}}}{p12}$$
Problem 1-15 (continued)

Solution: 
\[ q = h A (T_w - T_0) \]
\[ T_w - T_0 = 40^\circ C \]
\[ h = 3500 \, \text{W/m}^2 \cdot \text{K} \]
\[ q = (3500 \, \text{W/m}^2 \cdot \text{K}) (1750 \, \text{m}^2) (40^\circ C) \]
\[ A = \pi \Phi h = \pi (2.5 \times 10^2 \, \text{m})(3 \, \text{m}) \]
\[ q = 1.05 \times 10^4 \, \pi \, \text{W} \]
\[ q = 3.3 \times 10^4 \, \text{W} \]

\[ q = m \cdot c_p (T_e - T_i) \Rightarrow \Delta T = \frac{q}{m \cdot c_p} \]

\[ \Delta T = \frac{3.3 \times 10^4 \, \text{W}}{(0.5 \, \text{Kg}) (4180 \, \text{J/kg} \cdot \text{K})} = 16^\circ C \]

Problem 1-20

If the radiant flux from the sun is \( 1350 \, \text{W/m}^2 \), what would be its equivalent black body temperature?

Sketch:

Sun Surface

Assumptions: Black Body.

Equations:

\[ q = \sigma A T^4 \]

\[ \sigma = 5.669 \times 10^{-8} \, \text{W/m}^2 \cdot \text{K}^4 \]

Solution:

\[ T^4 = \frac{q}{\sigma A} = \frac{1350 \, \text{W/m}^2}{5.669 \times 10^{-8} \, \text{W/m}^2 \cdot \text{K}^4} \]

\[ T = 393 \, \text{K} \]

Problem 1-21

A 4.0 cm diameter sphere is heated to a temperature of 200°C and is enclosed in a large room at 20°C. Calculate the radiant heat loss if the surface emissivity is 0.6.

Sketch:

\[ T = 200^\circ C \]

\[ T_{ambient} = 20^\circ C \]

\[ \text{Radius} = 4\, \text{cm} \]
Problem 1-21 (continued)

Assumptions: Uniform temperature sphere, black body radiation, no other sources, large enclosure.

Equations:
$$ q = \varepsilon \sigma A_1 (T_1^4 - T_2^4) \\
A_1 = \frac{4}{3} \pi r^3 = \frac{16}{3} \left(64 \times 10^{-4} \text{ m}^2\right) \\
\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \\
\varepsilon = 0.6 $$

Solution:
$$ q = (0.6) \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}\right) \left(\frac{16}{3} \times 10^{-4} \text{ m}^4\right) \left[(473 \text{ K})^4 - (293 \text{ K})^4\right] $$

$$ q = \frac{36.6 \text{ W}}{7.6 \text{ W}} $$

Problem 1-27

A solar radiant heat flux of 700 $\frac{\text{W}}{\text{m}^2}$ is absorbed in a metal plate that is perfectly insulated on the back side. The convection heat transfer coefficient on the plate is $11 \frac{\text{W}}{\text{m}^2 \text{K}}$, and the ambient air temperature is $30^\circ \text{C}$. Calculate the temperature of the plate under equilibrium conditions.

Sketch

$$ \frac{q}{A} = 700 \frac{\text{W}}{\text{m}^2} \quad \Rightarrow \quad h = 11 \frac{\text{W}}{\text{m}^2 \text{K}} \quad T = 30^\circ \text{C} $$

Assumptions: Enclosed in large room, heat absorbed is equal to convection.

Equations:
$$ q = h A \left(T_w - T_\infty\right) \quad \text{[Eq. 1-8]} $$

Solution:
$$ \frac{q}{A h} = T_w - T_\infty \quad \Rightarrow \quad T_w = T_\infty + \frac{q}{A \cdot h} $$

$$ T_w = 30^\circ \text{C} + \frac{700 \frac{\text{W}}{\text{m}^2}}{11 \frac{\text{W}}{\text{m}^2 \text{K}}} = 30^\circ \text{C} + 63.6^\circ \text{C} = 93.6^\circ \text{C} \quad \text{[Ans]} $$
Problem 1-40

An ice-skating rink is located in an indoor shopping mall with an environment air temperature of 22°C and radiation surrounding walls of about 25°C. The convection heat transfer coefficient between the ice and air is about 10 W/m²K because of air movement and skaters' motion. The emissivity of the ice is about 0.95. Calculate the cooling required to maintain the ice at 0°C for an ice rink having dimensions of 12 m by 40 m. Obtain a value for the heat of fusion of ice and estimate how long it would take to melt 3 mm of ice from the surface if no cooling is supplied and the surface is considered isolated on the back side.

Sketch:

\[ T_o = 22°C \]
\[ T = 0°C \]

Assumptions: Steady State. Radiation transfer in enclosure AND Convection

Equations:

- \[ q_{rad} = eA(T_i^4 - T_e^4) \] 
- \[ q_{conv} = hA(T_i - T_o) \]

\[ \begin{align*}
A &= 12 \times 40 = 480 \text{m}^2 \\
T_i &= 22°C \text{ (enclosure)} \\
T_e &= 25°C \\
T_s &= 0°C \text{ (ice)}
\end{align*} \]

\[ e = 0.95, \quad h = 10 \frac{W}{m²K} \]

Solution:

\[ q = q_{rad} + q_{conv} = eA(T_i^4 - T_e^4) + hA(T_s - T_o) \]
\[ = (0.95)(480 \text{m}^2)(5.66 \times 10^{-8} \frac{W}{m²K})(273 K^4 - 298 K^4) + 10 \frac{W}{m²}(490 \text{m}^2)(0.95) \]
\[ = 6027.3 \text{ W} - 105600 \text{ W} \]
\[ q = -166 \text{ kW} \]

New Assumption: No Cooling \rightarrow non steady state.

New Equation:

\[ q = \frac{(\text{Volume}) \times \frac{\Delta H}{\rho}}{\Delta t} \]

Solution:

\[ \Delta t = \frac{12 \times 40 \times (3 \times 10^{-3} m^3) (3.348 \times 10^5 \frac{J}{kg})}{166 \text{ kW}} \]
\[ \Delta t = 3 \times 10^3 \text{ seconds} \]