A wall is constructed of a section of stainless steel, $k = 16 \, \text{W/m}^\circ\text{C}$, 4.0 mm thick with identical layers of plastic on both sides of the steel. The overall heat transfer coefficient, considering convection on both sides of the plastic, is $120 \, \text{W/m}^2\circ\text{C}$. If the overall temperature difference across the arrangement is $60^\circ\text{C}$, calculate the temperature difference across the stainless steel.

\[ T_2 - T_1 = -50^\circ\text{C} \]

\[ U = 120 \, \text{W/m}^2\circ\text{C} \]

Steady state, plane wall, $T = D$

**Equations**

\[ U = \frac{1}{\frac{1}{h_1} + \frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \frac{1}{h_2}} \]

where \( \Delta x_1 = 4.0 \, \text{mm} \)

\( \Delta x_2 = \text{thickness of plastic} \)

\( k_1 = 16 \, \text{W/m}^2\circ\text{C} \)

\( k_2 = \text{Plastic conductivity} \)

\( h_1, h_2 \) and convection

\[ q = \frac{U}{A} \Delta T_{\text{overall}} \quad \text{eq} \left[ 2-13 \right] \]

\[ q \div A = \frac{\Delta T_{\text{steel}}}{\Delta x_1} \]

\[ q \div A = \frac{\Delta T_{\text{steel}}}{\frac{\Delta x_1}{k_1}} \]

**Solution**

\[ q \div A = U \Delta T_{\text{overall}} = 120 \, \text{W/m}^2\circ\text{C} \times (-50^\circ\text{C}) \]

\[ q \div A = \frac{\Delta T_{\text{steel}}}{\Delta x_1} \div \frac{\Delta x_1}{k_1} = 120 \, \text{W/m}^2\circ\text{C} \times (-50^\circ\text{C}) \times \frac{4.0 \, \text{mm}}{16 \, \text{W/m}^2\circ\text{C}} \]

\[ \Delta T_{\text{steel}} = 15^\circ \text{C} - 1.8^\circ \text{C} \]
An ice chest is constructed of styrofoam \( k = 0.033 \, \text{W/m}^\circ\text{C} \) with inside dimensions of 25 cm x 40 cm x 100 cm. The wall thickness is 5 cm. The outside of the chest is exposed to air at 25°C with \( h = 10 \, \text{W/m}^2\circ\text{C} \). If the chest is completely filled with ice, calculate the time for the ice to completely melt. State your assumptions. \( H_f = 330 \, \text{kJ/kg} \)

**Sketch**

Assumptions: The inside wall temperature is 0°C. Steady state, plane wall, 1-D

\[
\rho = 0.033 \, \frac{\text{W}}{\text{m}^2\circ\text{C}}
\]

5 cm thick

\[
\begin{align*}
l &= 10 \, \text{W/m}^2\circ\text{C} \\
h &= 25^\circ\text{C}
\end{align*}
\]

Equations

\[
\bar{q} = \frac{T_1 - T_2}{\frac{1}{h_1} + \frac{A\chi}{k}}
\]

Solution

\[
\begin{align*}
\bar{q} &= \frac{V \rho H_f}{q} \\
\bar{q} &= \frac{V \rho H_f}{q} = \frac{(25 \text{ cm} \times 40 \text{ cm} \times 100 \text{ cm}) \frac{\text{m}^3}{10^6 \text{cm}^3} \frac{3 \text{ kg}}{10^3 \text{m}^3}}{10^6 \text{cm}^3} \\
V &= 25 \text{ cm} \times 40 \text{ cm} \times 100 \text{ cm} = 0.1 \, \text{m}^3
\end{align*}
\]

\[
A_o = 2(25 \text{ cm} \times 40 \text{ cm} + 40 \text{ cm} \times 100 \text{ cm} + 25 \text{ cm} \times 100 \text{ cm}) = 1.5 \, \text{m}^2
\]

\[
U = \frac{1}{h_1} + \frac{A\chi}{k} = \frac{1}{10^6 \text{cm}^2} + \frac{5 \times 10^{-5} \text{m}}{0.033 \, \text{W/m}^\circ\text{C}} = \frac{1}{0.1} + 1.5 = \frac{5 \, \text{W}}{0.8 \, \text{m}^2\circ\text{C}}
\]
Problem 2.14 continued

\[ \frac{q}{A_s} = h \Delta T \]
\[ t = \frac{V p H_f}{q} \]
\[ A_s = \text{surface area of rectangular box} \]
\[ A_{s,i} = \text{inside area} \]
\[ A_{s,o} = \text{outside area} \]
\[ q = A_s h \Delta T \]
\[ t = \frac{V p H_f}{A_s h \Delta T} \]
\[ \begin{align*}
V &= 0.1 \text{ m}^3 \\
\rho &= 10^3 \frac{\text{kg}}{\text{m}^3} \\
H_f &= 330 \frac{\text{kJ}}{\text{kg}} \\
L &= 6.25 \frac{\text{kJ}}{\text{kg}} \\
A_{s,i} &= 1.5 \text{ m}^2 \\
A_{s,o} &= 7(35 \times 50 \text{ cm} + 50 \text{ cm} \times 110 \text{ cm} + 35 \text{ cm} \times 110 \text{ cm}) \\
&= 2.22 \times 10^4 \text{ cm}^2 \\
&= 2.22 \text{ m}^2 \\
t &= 1.4 \times 10^5 \left( \frac{1.5 \text{ m}^2}{2.22 \text{ m}^2} \right) = 0.67 \times 10^5 \text{ s} \\
&= 11 \text{ days} \\
\end{align*} \]

So a reasonable calculation gives 10 to 16 days for the ice to melt.
Problem 2.15

A spherical tank, 1 m in diameter, is maintained at a temperature of 120°C and exposed to a convection environment. With \( h = 2.5 \text{ W/m}^2 \text{K} \) and \( T_\infty = 15\text{°C} \), what thickness of urethane foam should be added to ensure that the outer temperature of the insulation does not exceed 40°C? What percentage reduction in heat loss results from installing this insulation?

Sketch:

**Assumptions:** Steady state, 1-D spherical model. Foam application does not change \( h \).

**Equations:** Convection equation

\[
q = -K \left( \frac{T - T_i}{r_o - r_i} \right)
\]

\[
q = -K \left( \frac{1}{r_o} - \frac{1}{r_i} \right) T
\]

\[
A = 4\pi r^2
\]

\[
\rho = \frac{25 \text{ W}}{m^2 \text{K}}
\]

\[
\kappa = 2.5 \times 10^{-3} \text{ mK/W}
\]

**Solution:**

First find the initial heat flow

\[ q = hA(T - T_\infty) = 25 \frac{\text{W}}{\text{m}^2 \text{K}} \cdot 4\pi (0.5 \text{ m})^2 (120\text{°C} - 15\text{°C}) = 3125 \text{ W} \]

Now add insulation; From Table 2-1, \( \rho = 28 \) \( \kappa = 25 \frac{\text{W}}{\text{m} \text{K}} \times 10^{-3} \)

\[
q = -K \left( \frac{T - T_i}{r_o - r_i} \right)
\]

\[
q = -K \left( \frac{1}{r_o} - \frac{1}{r_i} \right) T
\]

\[
A = 4\pi r^2
\]

\[
\frac{\rho r^2}{2} - 2 \kappa r - 2 \kappa \kappa \frac{r^2}{\rho} = 0
\]

\[
\frac{r_o^2}{2} - 2 \kappa r - 2 \kappa \kappa \frac{r^2}{\rho} = 0
\]

\[
\frac{r_o^2}{r_i^2} = 3.2 \times 10^{-3}
\]

\[
\rho \frac{r_o^2}{2} - 2 \kappa r - 2 \kappa \kappa \frac{r^2}{\rho} = 0
\]

\[
\frac{r_o^2}{r_i^2} = 3.2 \times 10^{-3}
\]
\[ h \ A \left( T_0 - T_\infty \right) = -4\pi k \ \frac{T_0 - T_i}{\left( \frac{r_0}{r_i} - \frac{1}{r_i} \right)} \]

\[ A = 4\pi r^2 \]

\[ h \times \pi r_0^2 \left( T_0 - T_\infty \right) = -4\pi k \ \frac{T_0 - T_i}{\left( \frac{1}{r_0} - \frac{1}{r_i} \right)} \]

\[ \left[ r_0 \left( \frac{1}{r_0} - \frac{1}{r_i} \right) \right] = -k \ \frac{T_0 - T_i}{h \ \frac{T_0 - T_\infty}{r_0 - r_i}} \]

\[ r_0^2 - r_i \ r_0 - \frac{k}{h} \ \frac{T_0 - T_i}{T_0 - T_\infty} = 0 \]

\[ r_0^2 - 0.5 \ r_0 - \frac{25 \times 10^{-3}}{25} = 0 \]

\[ r_0^2 - 0.5 \ r_0 - 3.2 \times 10^{-3} = 0 \]

\[ r_0 = 0.50318 \quad \Rightarrow \ r_0 - r_i = 3.2 \times 10^{-3} \ m \]

\[ q = h \ A \left( T_0 - T_\infty \right)^2 \left( 25 \ \frac{w}{m^2 K} \right) \ \frac{4\pi (0.5)^2 (120 - 15)}{8} \approx 8.2 \ kW \]

\[ q \ \text{with insulation} = 25 \ \frac{w}{m^2 K} \ \frac{4\pi (0.503)^2 (40 - 15)}{8} = 2.0 \ kW \]

\[ \% \ \text{reduction} = \frac{q - q_i}{q} = \frac{8.2 - 2}{8.2} = 76\% \]
Problem 2-17

Suppose the sphere in problem 2-15 is covered with a 1-cm layer of an insulating material having \( K = 50 \text{ mW/m}^2\text{C} \) and the outside of the insulation is exposed to an environment having \( h = 20 \text{ W/m}^2\text{C} \) and \( T_0 = 10^\circ \text{C} \). The inside of the sphere remains at 100°C. Calculate the heat transfer under these conditions.

Sketch:

\[ T_0 = 10^\circ \text{C} \]

\[ h = 20 \text{ W/m}^2\text{C} \]

Assumptions:
- Inside of insulation is uniform 100°C
- Steady state
- 1D spherical heat conduction

Equations:

\[ q = hA(T_0 - T_0) \]

\[ Q = \frac{4\pi KT(T_1 - T_0)}{r_1 - \frac{1}{r_0}} \]

Solution:

\[ q = \frac{20}{m^2\text{C}} \]

\[ T_0 = T_1 + \frac{q}{4\pi KT} \left( \frac{1}{r_0} - \frac{1}{r_1} \right) \]

\[ T_0 = T_1 + \frac{q}{4\pi KT} \left( \frac{1}{r_0} - \frac{1}{r_1} \right) \]

\[ q = \frac{T_0 - T_1}{4\pi KT\left(\frac{1}{r_0} - \frac{1}{r_1}\right)} - hA \]

\[ q = \frac{100^\circ \text{C} - 10^\circ \text{C}}{4\pi \times 0.050 \text{ W/m}^2\text{C} \left(\frac{1}{0.05} - \frac{1}{0.5}\right)} - \frac{100^\circ \text{C} - 10^\circ \text{C}}{20 \text{ W/m}^2\text{C}} \]

\[ q = \frac{100^\circ \text{C} - 10^\circ \text{C}}{4\pi \times 0.050 \text{ W/m}^2\text{C} \left(\frac{1}{0.05} - \frac{1}{0.5}\right)} + \frac{100^\circ \text{C} - 10^\circ \text{C}}{20 \text{ W/m}^2\text{C} \left(\frac{1}{0.05} - \frac{1}{0.5}\right)} \]

\[ q = 94 \text{ W} \]
Problem 2-21
A 1.0 mm diameter wire is maintained at a temperature of 400°C and exposed to a convection environment at 40°C with \( h = 120 \text{ W/m}^2\text{K} \). Calculate the thermal conductivity which will just cause an insulation thickness of 0.2 mm to produce a "critical radius". How much of this insulation must be added to reduce the heat transfer by 75 percent from that which would be experienced by a bare wire?

**Sketch**

\[ T_{oo} = 40^\circ C \]
\[ h = 120 \text{ W/m}^2\text{K} \]

thickness = \( r_0 - r_i = 0.2 \text{ mm} \)

**Assumptions:**
- The insulated wire is modeled as a 1D cylinder.
- Metal temp is uniform.
- Steady state.

**Equations:**

\[ eq \{2-18\} \quad r_0 = \frac{K}{h} \]
\[ eq \{2-17\} \quad q = \frac{2\pi L(T_i - T_{oo})}{ln(r_i/r_c) + \frac{1}{K} + \frac{1}{r_oh}} \]

**Solution**

Critical thickness is \( r_0 = \frac{K}{h} \)

\[ r_0 = r_i + 0.2 \text{ mm} \]

\[ K = h(r_i) = h(r_i + 0.2 \text{ mm}) = 120 \text{ W/m}^2\text{K} \left( \frac{1.0 \text{ mm}}{2} + 0.2 \text{ mm} \right) \]

\[ = 84 \text{ W/m}^2\text{K} \]

\[ q(r_i) \quad r_0 = \frac{2\pi L(T_i - T_{oo})}{ln(r_i/r_c) + \frac{1}{r_0h}} \]

\[ q(r_i) \quad r_0 = 0.25 \quad \frac{2\pi L(T_i - T_{oo})}{ln(r_i/r_c) + \frac{1}{r_0h}} \quad = \quad \frac{0 + r_ih}{ln(r_i/r_c) + \frac{1}{r_0h}} \]

\[ r_0 = 0.1338 \text{ m} \quad \text{thickness} = 0.1333 \text{ m} \]
Problem 2-21, plot of eq [2-17]

$dT=400-40$, $ri=.0005$, $h=120$, $k=.084$
A cylindrical tank 80 cm in diameter and 2.0 m high contains water at 80°C. The tank is 90% full and insulation is to be added so that the water temperature will not drop more than 2°C per hour. Using the information given in the chapter, specify an insulating material and calculate the thickness required for the specified cooling rate.

Assumptions: thickness of insulation \( \ll \phi \), so plane wall equation

Steady state

Metal temperature is uniform and \( = 80°C \)

No convection. Environment temperature \( = 20°C \)

Equations:

\[
q = -kA \frac{dT}{dx} \quad \Rightarrow \quad \frac{q}{A} = -k \frac{dT}{dx}
\]

\[
\text{Area cylinder} = 2\pi r L = 2\pi (0.4)^2 (2)
\]

\[
\frac{dT}{dt} = \frac{q}{c_p \rho V}
\]

Solution: Use fiberglass insulation \( k = 33 \times 10^{-3} \text{ W/m°C} \)

Table 2-1

\[
\frac{dT}{dx} \frac{q}{c_p \rho V} = \frac{1}{2} \frac{k}{c_p \rho V} \frac{(0.8)^2 (0.7)}{10^{-3}} \frac{3600}{2 \text{ hours}} (80°C - 20°C)
\]

\[
\Delta x = \frac{-kA \frac{dT}{dx}}{\frac{1}{2} \frac{k}{c_p \rho V}} = \frac{1}{2} \frac{k}{c_p \rho V} \frac{(0.8)^2 (0.7)}{10^{-3}} \frac{3600}{2 \text{ hours}} (80°C - 20°C)
\]

\[
\Delta x = 5.8 \times 10^{-3} \text{ m}
\]

Only 6 mm so this matches our assumption that thickness \( \ll \phi \)
Problem 2-28

An insulation system is to be selected for a furnace wall at 1000°C using first a layer of mineral wool blocks followed by fiberglass boards. The outside of the insulation is exposed to an environment with \( h = 15 \text{ W/m}^2\text{C} \) and \( T_o = 40°C \). Using the data in Table 2-1 calculate the thickness of each insulating material such that the interface temperature is not greater than 400°C and the outside temperature is not greater than 55°C. Use mean values for the thermal conductivities. What is the heat loss in this wall in W/m²?

**Sketch:**

![Sketch of the insulation system]

\[ T_1 = 1000°C \]
\[ T_2 = 400°C \]
\[ T_3 = 55°C \]
\[ T_o = 40°C \]

**Assumptions:** Steady state, 1-D plane wall

\[ k \neq f(t) \]

**Equations:**

\[ q = \frac{T_1 - T_o}{\frac{\Delta x_A}{k_A} + \frac{\Delta x_B}{k_B} + \frac{1}{h}} = \frac{T_1 - T_2}{\frac{\Delta x_A}{k_A}} \]

**Solution:**

\[ \frac{T_1 - T_2}{\frac{\Delta x_A}{k_A}} = \frac{T_2 - T_3}{\frac{\Delta x_B}{k_B}} = \frac{\Delta x_A}{\frac{\Delta x_B}{k_B}} \]

\[ \Delta x_A = \frac{k_A}{h} \frac{T_1 - T_2}{T_3 - T_o} \]
\[ \Delta x_B = \frac{k_B}{h} \frac{T_2 - T_3}{T_3 - T_o} \]

\[ \Delta x_A = \frac{9.1 \times 10^{-3} \text{ W/m°C}}{15 \text{ W/m°C}} \left( \frac{1000 - 400}{55 - 40} \right) \]
\[ \Delta x_B = \frac{4.3 \times 10^{-3} \text{ W/m°C}}{15 \text{ W/m°C}} \left( \frac{400 - 55}{55 - 40} \right) \]

\[ \Delta x_A = 0.24 \text{ m} \]
\[ \Delta x_B = 0.066 \text{ m} \]

\[ q = \left( \frac{1000 - 400}{24 \text{ m}} \right) \frac{9.1 \times 10^{-3} \text{ W}}{\text{m}^2} = 2.28 \text{ W/m}^2 \]