Problem 3.35

For the insulated corner section shown in Figure 3.35, derive an expression for the nodal equation of node \( m, n \) under steady state conditions.

**Sketch**

\[ \Delta x \]
\[ \Delta y \]

**Equations**
\[ q = -KA \frac{\partial T}{\partial x} - KA \frac{\partial T}{\partial y} \]
\[ \frac{\partial T}{\partial x} = \frac{T_e - T_i}{x_L - x_i} \]
\[ \frac{\partial T}{\partial y} = \frac{T_e - T_i}{y_e - y_i} \]

\[ \varepsilon q = -KA \left[ \frac{T_{m,n} - T_{m+1,n}}{\Delta x/2} \right] - KA \left[ \frac{T_{m,n} - T_{m,n+1}}{\Delta y/2} \right] = 0 \]

\[ A_x = \frac{\Delta y}{2} \quad A_y = \frac{\Delta x}{2} \]

\[ q = -K \frac{\Delta y}{\Delta x} \left[ T_{m,n} - T_{m,n-1} \right] - K \Delta x \left[ \frac{T_{m,n} - T_{m+1,n}}{\Delta y} \right] = 0 \]

\[ \frac{\Delta y}{\Delta x} \left[ T_{m,n} - T_{m,n-1} \right] + \frac{\Delta x}{\Delta y} \left[ T_{m,n} - T_{m+1,n} \right] = 0 \]

\[ T_{m,n} = \frac{T_{m,n-1} \Delta x^2 + T_{m+1,n} \Delta y^2}{\Delta x^2 + \Delta y^2} \]

If \( \Delta x = \Delta y \) then, \( T_{m,n} \) simplifies to:

\[ T_{m,n} = \frac{T_{m,n-1} + T_{m+1,n}}{2} \]
Problem 3-50

The fin shown in Figure 3-50 has a base maintained at 300°C and is exposed to the convection environment indicated. Calculate the steady state temperatures of the nodes shown and the heat loss if \( k = 1.0 \text{ W/m°C} \).

**Sketch**

![Sketch of the fin with node temperatures](image)

**Assume:** Finite difference method

Equation for nodes 1, 2, 3, 4 are identical, nodes 5, 6, 7 are identical, node 4 is unique, Node 8 is unique.

**Energy balance:**

For nodes 5, 6, 7

\[
T_5 = \frac{2T_1 + T_6 + T_w}{10}
\]

For node 5, 6, 7

\[
\begin{align*}
\dot{q} &= k A_y \left( \frac{T_5 - T_6}{\Delta x} \right) + k A_x \left( \frac{T_5 - T_w}{\Delta y} \right) + k A_x \left( \frac{T_5 - T_1}{\Delta y} \right) \\
&T_5 \left[ \frac{A_y}{\Delta x} + \frac{A_y}{\Delta x} + \frac{2A_x}{\Delta y} \right] = T_6 \frac{A_y}{\Delta x} + T_w \frac{A_y}{\Delta x} + 2T_1 \frac{A_y}{\Delta x}
\end{align*}
\]

**Solution**

\[
T_5 = \frac{T_6 + T_w + 2T_1 \left( \frac{A_x}{A_y} \right)}{2 + 2 \left( \frac{A_x}{A_y} \right)}
\]

**Node 1, 2, 3**

\[
\begin{align*}
\dot{q} &= k A_y \left( \frac{T_2 - T_1}{\Delta x} \right) + k A_y \left( \frac{T_2 - T_3}{\Delta x} \right) + k A_x \left( \frac{T_2 - T_6}{\Delta y} \right) + h A_x \left( T_2 - T_0 \right) \\
&T_2 \left[ \frac{A_y}{2\Delta x} + \frac{A_y}{2\Delta x} + \frac{A_x}{\Delta y} + \frac{A_x}{\Delta y} \right] = (T_1 + T_3) \left[ \frac{k A_y}{2\Delta x} \right] + \frac{k A_x}{\Delta y} T_6 + h A_x T_0
\end{align*}
\]

\[
T_2 = \frac{T_1 + T_3 + \left( \frac{2k A_x}{2A_y} \right) T_6 + 2A_x A_y I}{2 + \frac{2A_x A_y}{A_y A_x} + \frac{2h A_x A_y}{k A_y}}
\]
\[ T_2 = \frac{T_1 + T_3 + 8T_6 + \frac{h H}{K} \Delta x}{2 + 8 + \frac{h}{k} 4 \Delta x} = \frac{T_1 + T_3 + 8T_6 + 32}{230} \]

For node 4: outside corner

\[ \sum q = k \frac{A_x}{2} \left( \frac{T_4 - T_8}{2} \right) + k \frac{A_y}{2} \left( \frac{T_4 - T_3}{2} \right) + h \left[ \frac{A_x}{2} + \frac{A_y}{2} \right] (T_4 - T_\infty) = 0 \]

\[ T_4 \left[ \frac{k \frac{A_x}{2}}{2 \Delta y} + \frac{k \frac{A_y}{2}}{2 \Delta x} + \frac{h}{2}(A_x + A_y) \right] = T_8 \frac{k \frac{A_x}{2}}{2 \Delta y} + T_3 \frac{A_y K}{2 \Delta x} + T_\infty h \left[ \frac{A_x}{2} \right] \]

\[ T_4 = T_8 + \frac{T_3 \left( \frac{\Delta y A_x}{\Delta x A_x} \right)}{1 + \frac{\Delta y A_x}{\Delta x A_x} + \frac{h}{k} \frac{\Delta y}{A_x} (A_x + A_y)} \]

\[ T_4 = T_8 + 4T_3 + 100 \frac{h}{k} \left[ 1 + \frac{1}{2} \right] \Delta y \]

\[ 1 + \frac{1}{4} \]

\[ T_4 = T_3 + T_8 \left( \frac{A_y}{\Delta y} \frac{A_x}{\Delta x} \right) + T_\infty h \left[ \frac{A_x \Delta x + A_y \Delta y}{A_y} \right] \]

\[ 1 + \frac{A_x A_x}{\Delta y \Delta x} + \frac{h}{k} \frac{A_x}{A_y} \frac{\Delta x}{\Delta y} \frac{A_x}{A_y} \]

\[ T_4 = T_3 + 4T_8 + 3T_8 \frac{h}{k} \Delta x \]

\[ 5 + 3 \frac{h}{k} \Delta x \]

\[ T_8 \]
Problem 3-550

Node 8

\[ \begin{align*}
C &= k \frac{A_x}{2} \left( T_8 - T_4 \right) + k A_y \left( \frac{T_8 - T_7}{\Delta x} \right) + h A_y \left( T_8 - T_\infty \right) = 0 \\
T_8 &= \frac{\frac{A_x}{A_y} \Delta x}{\frac{A_x}{A_y} \Delta y + 1 + \frac{h \Delta x}{K}} \left( T_4 + T_7 + \frac{h \Delta x T_\infty}{K} \right) \\
&= \frac{A_x}{A_y} \Delta x \frac{T_4 + T_7 + \frac{h \Delta x T_\infty}{K}}{5 + \frac{h \Delta x}{K}}
\end{align*} \]
Holamn, Problem 3-50

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NODE 1 (AND 2,3)
\[(A3+C3+B4+B3*8)J5*J3(J4*J5)/(10+4*J5*J3/J4)\]

NODE 5 (6,7)
\[(C4+A4+B3*8)/10\]

NODE 8
\[(D4+4*E3+F3*J3/J4*J5)/(5+J3/J4*J5)\]

NODE 4
\[(D3+4*E4+F3*J3/J4*J5)/(5+3*J3/J4*J5)\]
Problem 3.55

A rod having a diameter of 2 cm and a length of 10 cm has one end maintained at 200°C and is exposed to a convection environment at 25°C with h = 40 W/m²K. The rod generates heat internally at the rate of 50 MW/m³ and the thermal conductivity is 35 W/mK. Calculate the temperatures of the nodes shown in Figure P3-55 assuming one-dimensional heat flow.

**Sketch**

\[ A_{cs} = \pi r^2 \]

Nodes 1, 2, 3, 4

\[ \Delta q = K A_{cs} \left( \frac{T - T_1}{\Delta z} \right) + K A_{cs} \left( \frac{T - T_2}{\Delta z} \right) \frac{\partial q}{\partial z} A_{cs} \Delta z + 2\pi \Delta r A \Delta z \left( h \left( T - T_0 \right) \right) = 0 \]

\[ T = T_0 + \frac{2\pi r A_{cs} \Delta z}{K} \frac{h}{k} \frac{Q_{cs} \Delta z}{T_0} + \frac{\left( \Delta z \right)}{k} T_0 \]

**Node 5**

\[ \Delta q = K A_{cs} \left( \frac{T_5 - T_4}{\Delta z} \right) - \frac{\partial q}{\partial z} A_{cs} \Delta z + 2\pi r A_{cs} A \Delta z \left( h \left( T_5 - T_0 \right) \right) = 0 \]

\[ T_5 = T_4 + \frac{\left( \Delta z \right)}{k} \frac{h}{k} \left( 2\pi r A_{cs} A \Delta z + h \left( 2\pi r A_{cs} A \right) T_0 \right) \]

\[ T_5 = T_4 + \frac{\left( \Delta z \right)}{k} \frac{h}{k} \frac{h}{k} \left( 2\pi r A_{cs} A \Delta z + h \left( 2\pi r A_{cs} A \right) T_0 \right) \]

\[ T_5 = T_4 + \frac{\left( \Delta z \right)}{k} \frac{h}{k} \left( 2\pi r A_{cs} A \Delta z + h \left( 2\pi r A_{cs} A \right) T_0 \right) \]

\[ T_5 = T_4 + \frac{\left( \Delta z \right)}{k} \frac{h}{k} \left( 2\pi r A_{cs} A \Delta z + h \left( 2\pi r A_{cs} A \right) T_0 \right) \]
$$\frac{K16 + K14 \times K17 + (A3 + C3) \times K13}{2 \times K13 + K14}$$

$$\frac{K16 + E3 \times K13 + K15 \times K17}{K13 + K15}$$

Node 1,2,3,4

Node 5

Convection: 4.022708 3.27691 3.149707 3.059596 3.243301

Convection total: 16.75222

Q-dot

Generation total: 15.70796

Error? 0.00000

Graph with data points.
Problem 3.75

Gas vs. Electric

Water

$h = 1500 \text{ W/m}^2\text{K}$

K = 202 $\text{W/m}^2\text{K}$

$\varepsilon = 0.34$ 0.04  
(Table A-10)

\begin{align*}
\frac{KA (T_2 - T_1)}{\Delta x} + \frac{\partial}{\partial x} \left( \frac{A \rho c_p (T_2 - T_1)}{\Delta x} \right) + \frac{\partial}{\partial y} \left( \frac{\epsilon A (T^4 - T_0^4)}{\Delta y} \right) &= \frac{\partial T}{\partial t} + \frac{\partial}{\partial y} \left( \frac{\kappa}{\Delta y} \frac{\partial T}{\partial y} \right) \\
\end{align*}
8mm aluminum
no heat transfer on bottom of pan between elements
boiling water on top $h=1500$, $t_{-\text{inf}}=100$, $k(\text{AL})=202$

Max = 286°C
Min = 283°C
Holman Problem 3-95
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Holman Problem 3-95
A copper sphere initially at a uniform temperature $T_0$ is immersed in a fluid. Electric heaters are placed in the fluid and controlled so that the temperature of the fluid follows a periodic variation given by

$$T_0 - T_m = A \sin \omega t$$

where $T_m$ is the time-average fluid temperature, $A$ is the amplitude of temperature wave, $\omega$ is the frequency.

Derive an expression for the temperature of the sphere as a function of time and the heat transfer coefficient from the fluid to the sphere.

**Sketch**

Assumption:
- Temperature of sphere is uniform and constant on time scale, $\omega$

Equation:
- $q = hA_s(T - T_0)$
- $q = C_p \rho V \frac{dT}{dt}$

$$A_s = \frac{4}{3} \pi r^2 \quad \pi r^2$$

$$C_p \rho V \frac{dT}{dt} = hA_s(T - T_0) = hA_s(T - T_m - A \sin \omega t)$$

$$m \frac{dT}{dt} = hA_s \left( T - T_m - A \sin \omega t \right)$$

$$2 \pi \mu \frac{dT}{dt} = m \left( T - T_m - A \sin \omega t \right)$$

$$\frac{2 \pi}{\mu} \frac{dT}{dt} = m \left[ T - T_m - A \sin \omega t \right]$$

$$\frac{2 \pi}{\mu} \frac{dT}{dt} = m \left[ T - T_m - A \sin \omega t \right]$$

$$T = T_m + \frac{A \omega^2}{h^2 + \omega^2} \left[ m \sin \omega t + w \cos \omega t \right] + C_1 e^{-\mu t}$$

Therefore:

$$T_0 = T_m + \frac{A \omega^2}{h^2 + \omega^2} \left[ m \sin \omega t + w \cos \omega t \right] + C_1 e^{-\mu t}$$
Problem 4-1
A=10, Tm=0, T0=15, m=.1, w:
Problem 4-3

What error would result from using the first few terms of equation 4-3 to compute the temperature at $x=0$ and $x=L$:

$$\frac{\Theta}{\Theta_i} = \frac{T - T_i}{T_c - T_i} = \frac{4}{\pi} \sum_{n=1,3,5,...} \frac{1}{n} e^{-\frac{n^2 \pi^2 t}{L^2}} \sin \left( \frac{n \pi x}{L} \right)$$

$x = 0$

$$\frac{T - T_i}{T_c - T_i} = \frac{4}{\pi} \sum_{n=1} \frac{1}{n} e^{-\frac{n^2 \pi^2 t}{L^2}} \sin \left( \frac{n \pi x}{L} \right)$$

Error:

$$\text{Error} = \frac{T_{\text{computed}} - T_{\text{actual}}}{T_{\text{actual}}}$$

$$T_{\text{computed}} = \frac{4}{\pi} \left( \frac{1}{1} \sin \frac{\pi}{2} + \frac{1}{3} \sin \frac{3\pi}{2} + \frac{1}{5} \sin \frac{5\pi}{2} + \frac{1}{7} \sin \frac{7\pi}{2} \right)$$

$$= \frac{4}{\pi} \left( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \right) = 0.92$$

Error:

$$\frac{1 - 0.92}{1} = 7\%$$
Problem 4-5

Sketch

Assumptions:
- Lumped heat capacity
- Heat transfer is 1-D
- Plane wall geometry

Equations:
\[ q = C_p \rho V \frac{dT}{dx} \]
\[ q = -k A \frac{dT}{dx} = k A S \Delta T \]
\[ S = \frac{A}{L} \quad A = 20 \times 20 \text{cm}^2 \]
\[ L = \frac{5 \text{cm}}{2} = 2.5 \text{ cm} \]
\[ \Delta T = T - T_w \]

Solution:
\[ q = C_p \rho V \frac{dT}{dx} = k S \Delta T \]
\[ \frac{1}{k} \frac{dT}{dx} = \frac{k A}{C_p \rho V L} \]
\[ \ln \left( \frac{T - T_w}{T_0 - T_w} \right) = -\frac{AK}{C_p \rho V L} x + C_1 \]
\[ T - T_w = C_2 e^{-\frac{AK}{C_p \rho V L} x} + C_1 \]

At \( x = 0 \):
\[ T = T_0 = 200 \text{°C} \]
\[ T - T_w = C_2 e^{0} + C_1 \]

\[ \frac{T - T_w}{T_0 - T_w} = \frac{-2KA}{C_p \rho V L} \]
\[ V = AL \]
\[ T = T_w + (T_0 - T_w) e^{\frac{-KA}{C_p \rho V L}} \]

\[ \frac{90 - 35}{200 - 35} = e \]
\[ m^2 = \frac{1}{2} \left( \frac{55}{165} \right) = \ln \left( \frac{1}{3} \right) \]
Problem 4.140

Oranges, with a diameter of about 3 in or 12 cm to be cooled from room temperature of 25°C to 3°C using an air-convection environment with $h = 45 \text{ W/m}^2\text{K}$ and $T_{oo} = 0°C$. Assuming that the oranges have the properties of water at 10°C, calculate the time required for the cooling and the total cooling required for 100 oranges.

**Sketch**

$$T = 25°C$$
$$r = 0.04 \text{m}$$
$$h = 45 \text{ W/m}^2\text{K}$$
$$T_{oo} = 0°C$$

**Assumptions:**

- Lumped heat capacity
- $T_{oo}$ is constant
- Oranges are a sphere
- $c_p = 4.179 \text{ kJ/kg°C}$
- $\rho = 997 \text{ kg/m}^3$
- $V = \frac{4}{3} \pi r^3$
- $A = 4 \pi r^2$
- $k = 0.604 \text{ W/m°C}$

Lumped heat capacity is not a good assumption, but lets try it anyway.

$$\frac{T - T_{oo}}{T_0 - T_{oo}} = e^{-\frac{hA}{kcpV}}$$

$$\frac{3 - 0}{25 - 0} = e^{-\frac{hA}{kcpV}}$$

$$2 = e^{-\frac{hA}{kcpV}}$$

$$hA = 3.06 \text{ W/m}^2\text{K}$$

$$\frac{hA}{kcpV} = \frac{45 \text{ W/m}^2\text{K}}{997 \text{ kg/m}^3 \cdot 4.179 \text{ kJ/kg°C} \cdot (0.04 \text{ m})^3 \cdot 1000 \text{ J/kg°C} \cdot 1.5} = -8.1 \times 10^{-4} \text{ s}^{-1}$$

$$T = \frac{1}{1.5} = 2.6175$$
Problem 8AGG 4-140

Orange #1 problem
Lumped heat capacity doesn't work because

\[
\frac{h \sqrt{\frac{a}{k}}}{}
\]

is greater than 0.1

Instead, use Heisler charts. For sphere, use chart in Figure 4-9, p 146

\[
\frac{k}{h_{f0}} = \frac{0.604 \frac{\text{W}}{\text{m} \cdot \text{C}}}{} \cdot (0.04) = 0.336
\]

\[
\frac{\Theta_0}{\Theta_i} = \frac{(T_0 - T_{\infty})}{(T_i - T_{\infty})} = \frac{3 - 0}{25 - 0} = \frac{3}{25} = 0.12
\]

From chart \( F_0 = \frac{\alpha \ell}{r_0} = 1.375 \)

\( \alpha \) where is \( 1.4 \times 10^{-7} \)

\( \ell = \frac{(1.375)(0.04)}{1.4 \times 10^{-7} \frac{\text{m}^2}{\text{s}}} = 4285 \text{ sec} \)

[Holman: 3888 sec]

From Figure 4-16, p 150

\( F_0 \beta_i^2 = \frac{\ell^2 \alpha \ell}{k'} = \frac{(45 \frac{\text{W}}{\text{m}^2})}{\left(0.604 \frac{\text{W}}{\text{m}^2}\right)^2} \left(1.4 \times 10^{-7} \frac{\text{m}^2}{\text{s}}\right)(4285 \text{ sec}) = 3.33 \)

\( \frac{h_{f0}}{l} = 3.0 \)

\( \frac{Q}{Q_0} = \frac{1.78 \times 0.9}{1} = 1.58 \)

\( Q = 2.5 \times 10^6 \text{ J} \)