# 4.7
The dark closed chamber has a small hole in a wall 3.0 m up from the floor. Once a year a beam of light enters the hole and strikes the polished mirror on the floor 4.0 m from the wall and reflects of it, lighting up a diamond embedded in forehead of the statue 20 m from the wall. How tall is the statue?

**Solution**

\[
\theta_i = \theta_r \quad \Rightarrow \quad \phi \quad \text{is the same in both triangles.}
\]

\[
\tan \phi = \frac{3}{4} \quad \Rightarrow \quad \frac{x}{16} = \tan \phi = \frac{3}{4}, \quad \text{or} \quad x = 12 \, \text{m}. \quad \text{or} \quad x = 12 \, \text{m.}
\]

# 4.20
A narrow beam of white light is incident at 60° on a sheet of glass 10.0 cm thick in air. The index of refraction for the red light is 1.505 and for violet light is 1.545. Determine the approximate diameter of emerging beam.

**Solution**

The diameter of an emerging beam is given by a difference of in horizontal displacement of red and violet light by the formula

\[
D = BC \cos \theta_i.
\]

The values of horizontal displacements of the red and violet rays are given by the formulas.
The difference

\[ h_v = AB = t \tan \theta_{i,v}. \]

\[ h_r - h_v = BC = t \left( \tan \theta_{i,r} - \tan \theta_{i,v} \right). \]

The angles of refraction for red and violet are given by the Snell’s law

\[ \theta_{i,r} = \arcsin \left( \frac{\sin \theta_i}{n_r} \right), \quad \theta_{i,v} = \arcsin \left( \frac{\sin \theta_i}{n_v} \right). \]

Calculations of these angles give the following values

\[ \theta_{i,r} = \arcsin \left( \frac{\sqrt{3}}{2 \cdot 1.505} \right) = 35.1^\circ, \quad \theta_{i,v} = \arcsin \left( \frac{\sqrt{3}}{2 \cdot 1.545} \right) = 34.1^\circ. \]

Finally, the diameter of the beam

\[ D = (10.0 \text{ cm}) \left[ \tan \left( 35.1^\circ \right) - \tan \left( 34.1^\circ \right) \right] \cos (60^\circ) = (10.0 \text{ cm}) \left[ 0.703 - 0.677 \right] (0.5) = 0.13 \text{ cm}, \]

\[ D = 0.13 \text{ cm}. \]

# 4.23

Light is incident in the air on air-glass interface. If the index of refraction is 1.7 find such incident angle that corresponding transmission angle is equal to \( \theta_i/2 \).

**Solution**

The Snell’s law

\[ n_a \sin \theta_i = n_g \sin \theta_t. \]

In our case \( n_a = 1 \), and \( \theta_i = \theta_i/2 \) or \( \theta_i = 2 \theta_t \). Thus,

\[ \sin 2 \theta_t = n_g \sin \theta_t, \quad \Rightarrow \quad 2 \sin \theta_t \cos \theta_t = n_g \sin \theta_t, \quad \Rightarrow \quad \cos \theta_t = n_g / 2. \]

\[ \theta_t = \arccos \left( \frac{n_g}{2} \right) = \arccos \left( \frac{1.7}{2} \right) = 31.8^\circ, \quad \theta_i = 63.6^\circ. \]
A coin is resting on the bottom of a tank of water (n = 1.333) 1.00 m deep. On the top of the water floats a layer of benzene (n = 1.50), which is 20.0 cm thick. Looking down nearly perpendicularly, how far beneath the topmost surface does the coin appear?

**Solution**

The depth of the coin image is \( AC \). From the triangle CAE

\[
\frac{AE}{AC} = \tan \theta_{i2}.
\]

From the triangle DAE

\[
\frac{AE}{AD} = \tan \theta_i, \quad AD = h_w + h_b.
\]

Thus,

\[
\frac{AC}{(h_w + h_b)} = \frac{\tan \theta_i}{\tan \theta_{i2}} = \frac{\sin \theta_i \cos \theta_{i2}}{\sin \theta_{i2} \cos \theta_i}.
\]

Using the Snell’s law we have

\[
n_w \sin \theta_i = n_b \sin \theta_{i1},
\]

and

\[
n_b \sin \theta_{i1} = n_{air} \sin \theta_{i2}.
\]

From these equations we have

\[
n_w \sin \theta_i = n_{air} \sin \theta_{i2}, \quad \text{and} \quad \frac{\sin \theta_i}{\sin \theta_{i2}} = \frac{n_{air}}{n_w} = \frac{1}{n_w}.
\]

Thus,

\[
AC = \frac{(h_w + h_b) \cos \theta_{i2}}{n_w \cos \theta_i}, \quad \text{and at } \theta_i \to 0, \quad AC = \frac{(h_w + h_b)}{n_w}.
\]

\[
AC = \frac{1.2 m}{1.33} = 0.902 m, \quad AC = 0.9 m.
\]
# 4.40

A beam of light strikes the surface of a smooth piece of plastic having an index of refraction of 1.55 at an angle with the normal 20.0°. The incident beam has component E-field amplitude parallel and perpendicular to the plane-of incidence of 10.0 \( V/m \) and 20.0 \( V/m \), respectively. Determine the corresponding reflected field amplitudes.

**Solution**

First we have to find the transmitting angle

\[
n_{\text{air}} \sin \theta_t = n_{\text{pl}} \sin \theta_i, \quad \text{or} \quad \theta_t = \arcsin \left( \frac{\sin(20.0^\circ)}{1.55} \right),
\]

\[
\theta_t = 12.75^\circ.
\]

Amplitudes of reflected wave are given by the formulas

\[
\left( \frac{E_r}{E_i} \right)_\parallel = r_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)},
\]

\[
\left( \frac{E_r}{E_i} \right)_\perp = r_\perp = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}.
\]

The values of reflected amplitude components are

\[
(E_r)_\parallel = (10.0 \ V/m) \frac{\tan(7.25^\circ)}{\tan(32.75^\circ)} = (10.0 \ V/m) \frac{0.127}{0.643} = 1.98 \ V/m,
\]

\[
(E_r)_\parallel = 1.98 \ V/m.
\]

\[
(E_r)_\perp = -(20.0 \ V/m) \frac{\sin(7.25^\circ)}{\sin(32.75^\circ)} = -(20.0 \ V/m) \frac{0.126}{0.541} = -4.6 \ V/m,
\]

\[
(E_r)_\perp = -4.7 \ V/m.
\]