Problem 1

Given \( G(s) = \frac{1}{s(s+1)(s+2)} \) \( H(s) = 1 \)

1. Sketch the root locus.
2. Find the break away pt.
3. Find the Jw-Axis crossing pt. & gain using Routh-Hurwitz
4. Find the Jw-Axis crossing pt. & gain using Li-Li criteria
5. Assuming a 2nd order dominant sys. Find location & gain where the 2nd order poles have a \( \tau = 0.05 \) \( (s=0.707) \) using \( \& \# 11 \) criteria
6. Assuming a 2nd order dominant sys. Find the location & gain where the system has a settling time of 10 sec using \( \& \# 11 \) criteria
7. What is the steady state error to a unit step input.

1. \( V_A = \frac{-1-2}{3} = -1 \), \( \theta_A = \pm 60\degree \ 180\degree 

2. Asymptotes & sketch

\[
2) \quad K = \frac{1}{GH} = \frac{1}{\frac{GWH}{GWH} \cdot \frac{GWH}{GWH}} = \frac{K}{1} = \frac{s^3 + 3s^2 + 2s}{s} = \frac{3s^2 + 6s + 2}{s}
\]

\[
\frac{dK}{ds} = 3s^2 + 6s + 2 = 0
\]

\[
\text{roots are } -1.577, -0.423
\]

Valid pt

3. For Routh-Hurwitz = need \( T(s) \)

\[
T(s) = \frac{KG}{1+KG} = \frac{K}{\frac{5(s+1)(s+2)}{1 + \frac{5}{s(s+1)(s+2)}}} = \frac{K}{\frac{5s^3 + 3s^2 + 2s + K}{s}}
\]

\[
\theta(s) = s^3 + 3s^2 + 2s + K
\]

\[
\begin{array}{c|ccc}
\lambda & 1 & 2 & K \\
\hline
s^3 & 1 & 3 & K \\
S^2 & 3 & K \\
S^1 & b_1 & K \\
S^0 & K &
\end{array}
\]

\[
b_1 = \left| \begin{array}{cc}
1 & 2 \\
3 & K
\end{array} \right| = \frac{1}{3}(K-6) = \frac{6-K}{3}
\]

Complex crossing pt where \( b_1 = 0 \)

\[
K = 6
\]

\[
\text{roots are } s^3 + 3s^2 + 2s + 6
\]

\[
-3, \pm j1.4142
\]
Problem 1 Cont

5) Locate P.T. Using K Criterion
   \[ \Theta_2 - \Theta_0 = \pm 180^\circ + n 360^\circ \]
   \[ \Theta_{12} = 90^\circ \]
   \[ \Theta_{23} = \tan^{-1}(\frac{y}{1}) \]
   \[ \Theta_{31} = \tan^{-1}(\frac{y}{2}) \]

   \[ -(\Theta_1 + \Theta_2 + \Theta_3) = -180 \]
   \[ -(90 + \tan^{-1}(y) + \tan^{-1}(\frac{y}{2})) = -180 \]

   Solve for \( y = 1.4142 \)

   \[ K = \frac{\pi}{2} \text{ pole lengths} \]
   \[ K = \frac{\pi}{2} \text{ BNL lengths} \]

   \[ K = \frac{\sqrt{y^2 + 1^2} \sqrt{y^2 + 2^2}}{1} = 6 \]

6) \( T_5 = 16 = \frac{y}{6} \)
   \[ v = \frac{4}{12} = \frac{1}{3} \]

   Locate Pole Using K Criterion
   \[ \Theta_2 - \Theta_0 = \pm 180^\circ + n 360^\circ \]
   \[ \Theta_{12} = 180 - \tan^{-1}(0.25) \]
   \[ \Theta_{23} = \tan^{-1}(\frac{y}{1.25}) = \tan^{-1}(0.75) \]
   \[ \Theta_{31} = \tan^{-1}(\frac{y}{2.25}) = \tan^{-1}(1.75) \]

   \[ -(180 - \tan^{-1}(\frac{y}{0.25}) + \tan^{-1}(\frac{y}{0.75}) + \tan^{-1}(\frac{y}{1.75})) = \pm 180 + n 360 \]
   \[ + \tan^{-1}(\frac{y}{0.25}) - \tan^{-1}(0.75) - \tan^{-1}(1.75) = 0 \pm n 360 \]

   Solving for \( y = 0.8292 \)

   \[ K = \sqrt{y^2 + 0.25^2} \sqrt{y^2 + 0.75^2} \sqrt{y^2 + 1.75^2} \]
   \[ = 1.87 \]

7) Unity FB. Type I sys. \( \epsilon_{55} = 0 \)
Problem 2

Given: \( G(s) = \frac{1}{(s+2)(s+4)} \)
\( H(s) = s+20 \)

1) Sketch the Root Locus

2) Assuming a 2nd order sys. find the location of gain where the system has a settling time of 0.25s using 1.1 % criteria.

3) Assuming a 2nd order sys. find the 2 locations of gains where the system has a time to peak of 0.5 seconds using the 1.1 % criteria.

4) What difference in the system performance would you expect from the 2 different CL pole locations found in part 3. Simulate. What problems do both systems have?

1) Asymptote 180°

\[ T_s = 0.25 = \frac{4}{6} \Rightarrow \theta = \frac{4}{0.25} = 16 \]

To find pole location use 4 crit.

\[ \theta_p = 180 - \tan^{-1} \left( \frac{y}{16-2} \right) = 180 - \tan^{-1} \left( \frac{4}{14} \right) \]

\[ \theta_2 = 180 - \tan^{-1} \left( \frac{4}{16-4} \right) = 180 - \tan^{-1} \left( \frac{4}{12} \right) \]

\[ \theta_2 = \tan^{-1} \left( \frac{4}{20-16} \right) = \tan^{-1} \left( \frac{4}{4} \right) = 0 \]

\[ \tan^{-1} \left( \frac{4}{4} \right) = \tan^{-1} \left( \frac{4}{4} \right) = 0 \]

\[ \tan^{-1} \left( \frac{4}{4} \right) + \tan^{-1} \left( \frac{4}{4} \right) = 180 \]

\[ y < 16.5 \]

\[ K = \frac{\sqrt{4^2 + 14^2} + \sqrt{4^2 + 12^2}}{\sqrt{4^2 + 4^2}} \]

\[ K = 26 \]
PROBLEM 2 Cont.

31 \( T_p = 0.5 = \frac{\pi}{4\omega} \)
\[ \omega = \frac{\pi}{\alpha_5} = 2\pi \]
\[ \omega = 6.28 \]

\( C_p \) poles:
- \( x_1 + j\omega_1 \)
- \( x_2 + j\omega_1 \)

To find \( x_1 \) & \( k_1 \), use \( \text{Eqn. 17} \).
To find \( k_1 \),
\[ \theta_{21} = \left( \theta_{p1} + \theta_{p2} \right) = \pm 180 \pm n360 \]
\[ \theta_{21} = \tan^{-1}(\frac{6.28}{20-x}) \]
\[ \theta_{p1} = 180 - \tan^{-1}(\frac{6.28}{x_1-2}) \]
\[ \theta_{p2} = 180 - \tan^{-1}(\frac{6.28}{x_1-4}) \]
\[ \frac{\tan^{-1}(6.28) - (180 - \tan^{-1}(\frac{6.28}{x_1-2}) + 180 - \tan^{-1}(\frac{6.28}{x_1-4}))}{\tan^{-1}(\frac{6.28}{x_1-2}) + \tan^{-1}(\frac{6.28}{x_1-4})} = \pm 180 \pm 360 \]

Silver for \( k_1 \)

\[ y_1 = 4.23 \leq \]
\[ k_1 = \frac{\sqrt{6.28^2 + (x_1-2)^2}}{\sqrt{6.28^2 + (x_1-4)^2}} \leq 2.46 > k_1 \]

To find \( x_2 \) & \( k_2 \)
\[ \theta_{22} = (\theta_{p3} + \theta_{p4}) = \pm 180 \pm n360 \]
\[ \theta_{22} = 180 - \tan^{-1}(\frac{2\pi}{x_2-20}) \]
\[ \theta_{p3} = 180 - \tan^{-1}(\frac{2\pi}{x_2-2}) \]
\[ \theta_{p4} = 180 - \tan^{-1}(\frac{2\pi}{x_2-4}) \]
\[ 180 - \tan^{-1}(\frac{2\pi}{x_2-20}) - (180 - \tan^{-1}(\frac{2\pi}{x_2-2}) + 180 - \tan^{-1}(\frac{2\pi}{x_2-4})) = \pm 180 \pm 360 \]
\[ \frac{-\tan^{-1}(\frac{2\pi}{x_2-10}) + \tan^{-1}(\frac{2\pi}{x_2-2}) + \tan^{-1}(\frac{2\pi}{x_2-4})}{\tan^{-1}(\frac{2\pi}{x_2-2}) + \tan^{-1}(\frac{2\pi}{x_2-4})} = 780 \leq \]
\[ x_2 = 35.7 \leq \]
\[ k_2 = \frac{\sqrt{6.72^2 + (x_2-2)^2}}{\sqrt{6.72^2 + (x_2-20)^2}} \leq 65.5 = k_2 \]
Problem 2 Cont.

4.)

Case 1  CL Poles \( \Rightarrow -4.23 \pm j 6.28 \)
CL ZEROS \( \Rightarrow \) NONE  (CL ZEROS ARE ZEROS OF G & POLES OF H)

Case 2  CL Poles \( \Rightarrow -35.7 \pm j 6.28 \)
CL ZEROS \( \Rightarrow \) NONE

The system is purely 2nd order
- Would expect faster \( T_s \) in Case 2
  Case 1  \( T_s = \frac{4}{4.23} = 0.945 \text{ sec.} \)
  Case 2  \( T_s = \frac{4}{35.7} = 0.112 \text{ sec} \)
- Would expect same \( T_p \)
- Would expect less OS in Case 2
  \( S = \cos(\theta) \)  \( \theta = \tan^{-1}(\frac{6.28}{4.23}) \)
  \( \theta_1 = \tan^{-1}(\frac{6.28}{4.23}) = 56^\circ \)
  \( S_1 = 0.6559 \)
  \( \theta_2 = \tan^{-1}(\frac{6.28}{35.7}) = 9.98^\circ \)
  \( S_2 = 0.984 \)

Both have a significant problem w/SS. Error
Problem 3.

\[ G(s) = \frac{1}{5(s^2 + 2s + 2)} \quad H(s) = \frac{1}{s + 3} \]

1) Plot R. I. in MATLAB
2) Find the break away pt. \( \Rightarrow \) see plots.
3) Find the jω-axes pt & gain

4) Find the pole locations (all 4) & gain when the dominant 2nd order poles have \( s = 0.5 \Rightarrow 7005 = 16.3\% \) is the 2nd order assumption valid?

Dominant CL poles have \( s = 0.5 \pm 0.91 \pm j0.71 \) with a gain = 2.61

Non-dominant poles @ -1.39 & -2.77

The nearest pole is only \( 3.4 \times \) real part of dominant pole so there will be some 1st order effect.

There will also be a CL zero @ -3

Expected: \( T_S = \frac{4}{0.41} = 10 \text{sec} \)
\[ T_P = \frac{\pi}{0.71} = 4.42 \text{ sec} \]
\[ 7005 = 16.3\% \]

Actual: \( T_S = 10.1 \)
\[ T_P = 3.39 \]
\[ 7005 = 12.8\% \]
3) Find the pole locations & gain for a settling time of 1 sec. Will the system achieve this?

\[ T_s = \frac{1}{\frac{4}{\tau}} \quad \tau = 4 \]

\[ K = 211 \quad \rightarrow \text{poles} \quad -4 \pm j2.59 \]

\[ +1.5 \pm j2.65 \quad \text{YIKES!} \]

\[ \tau \text{ at } 0 \times e^{-3} \quad \text{UNSTABLE} \]

No, system will be unstable & will never settle.
Problem 3 parts 1-3

Problem 3 part 4
Problem 3 part 4 – Step Response

Problem 3 part 5
Problem 4

Given \( G(s) = \frac{5s+1}{s^2(s+9)} \) \( H(s) = 1 \)

1) Sketch by hand - see if you can come up with 2 possibilities for the root locus

(A)

2) Asymptotes \( \theta_a = \pm 90 \)

\[ \Gamma_a = \frac{-9 - (-1)}{2} = \frac{-8}{2} = -4 \]

→ Could tell which one happens by calculating break-away/in pts.

→ No valid pts → (A)

→ Valid pts → (B)

2) Plot in MATLAB

3) Find break-away pt  \( \frac{3}{5} \) → See plot

4) \( T_p = 0.5 \cdot \frac{\pi}{\omega_d} \) \( \omega_d = \frac{\pi}{0.5} = 2\pi \)

\[ K = 64 \]

CL poles = \(-3.91 \pm j6.29\), -1.17 \( \leftarrow \) will get some pole/zero cancellation

CL zero = -1

EXPECTED

Based on 2nd order
Assumed

\[ T_p = 0.5 \text{ sec} \]

\[ T_s = \frac{4}{3.91} = 1.02 \text{ sec} \]

\[ \theta = \tan^{-1}\left(\frac{6.28}{3.91}\right) = 58.1^\circ \]

\[ \gamma = \cos\theta = 0.528 \]

\( 70\% \) of \( 0.5 \) = 14.29\%

ACTUAL

\[ T_p = 0.5 \text{ sec} \]

\[ T_s = 1.93 \text{ sec} \]

\( 70\% \) of 0.5 = 27.77\%

From effect of zero at -1
5) Find the gain for a critically damped sys.

- Find all 3 pole locations.
- Simulate \( \Rightarrow \) Does sys. perform like critically damped sys.? Why or why not.

\[ \text{K} = 27, \quad \text{CL poles: } -3, -3, -3 \]
\[ \text{CL zero: } @ -1 \]

\( \Rightarrow \) For critically damped sys. would expect no overshoot.
- System has 29.97% YIKES!
- Comes from the effect of the zero @ -1.

See plot.

6) What is the steady-state error to a unit step input?

\( \Rightarrow \) Unity F.B. Type 2 sys.

\[ \text{CS} = 0 \]
Problem 4 – Parts 2-3, 5

Problem 4 – Part 4
Problem 4

System: syst
Peak amplitude: 1.25
Overshoot (%): 24.9
At time (sec): 0.989

Problem 4 part 5 – Step Response