ME 402/502 - Gas Dynamics

Solution to Example – C-D Nozzle Performance

Ref: Fig 3.14; 3.15; and the complete picture is on page 132; Fig. 4.4, and material that follows. We will eventually develop the NSW and other shockwave concepts. In this solution, all ratios are found from Comprop2.

\[ \text{Given: } \gamma = 1.4 \]
\[ A_1 = 10 \text{ cm}^2 \]
\[ P_1 = 250 \text{ kPa} \]
\[ V_1 = 80 \text{ m/sec} \]
\[ T_1 = 28^\circ \text{C} = 301.15 \text{ K} \]

\[ \text{Find: } P_0, \text{ stagnation pressure} \]
\[ \text{Find: } \frac{P_0}{P_1} = 2.100 \]
\[ \frac{P_0}{P_1} = 725.9165 \]
\[ \frac{P_0}{P_1} = 1.037 \]

\[ \frac{P_0}{P_1} = 1.037 \]
\[ \frac{T_0}{T_1} = 1.01 \]

\[ \frac{T_0}{T_1} = 0.2284 \]
\[ \frac{T_0}{T_1} = 304.16 \text{ K} \]

\[ \frac{A_1}{A^*} = 2.614 \]
\[ A^* = \frac{10 \text{ cm}^2}{2.614} = 3.8256 \text{ cm}^2 \]

\[ \frac{P_2}{P_1} = \frac{(P_0/P_1) \to M_1}{(P_0/P_2) \to M_2} = \frac{250}{700} = 0.3571 \]

\[ \frac{1.037}{0.3571} = \frac{(P_0/P_2)}{M_2} \]

\[ \frac{P_0}{P_2} (M_2) = 2.9036 \]

\[ M_2 = 1.334 \]

Discuss which paths are?
Cont'd:  
\[
\frac{M_2}{1.334} = \frac{T_2}{T_o} = 1.356
\]
\[
T_2 = \frac{304.16}{1.356} = 224.31 \text{ K}
\]
\[
a_2 = \sqrt{1.4 \times 287 \times 224.31} = 300.21 \text{ m/sec}
\]
\[
V_2 = a_2 \times M_2 = 400.48 \text{ m/sec}
\]

"Check:"  
\[
\frac{P_o}{P_2} = 2.904; \quad P_2 = 249.97 \text{ kPa}
\]
\[
\frac{A_2}{A^*} = 1.082
\]
\[
\therefore A_2 = 1.082 \times 3.8256 = 4.1393 \text{ cm}^2
\]
\[
\frac{A^+}{A^*} = 1.2 \quad \frac{P_o}{P^*} = 1.893; P^* = 383.47 \text{ kPa}
\]
\[
T^* = 8333 \times 304.16 = 253.46 \text{ K}
\]
\[
a^* = \sqrt{1.4 \times 287 \times 253.46} = 319.12 \text{ m/sec}
\]

What is the result if we look at the Subsonic Branch for the diverging section?