I. Two charges $q_1 = 10.0 \mu C$ and $q_2 = -10.0 \mu C$ are separated by the distance $2d = 20.0 \text{ cm}$ as shown in the figure. Find (a) the electric field at the point $P$ ($b = 20 \text{ cm}$); (b) the value of the charge $Q$ that has to be placed at the point $D$ ($h = 35 \text{ cm}$) to compensate the electric field created by the charges $q_1$ and $q_2$ at the point $P$.

Solution

a) The field due to a point charges $q_1$ and $q_2$ at the point $P$ is the sum of two vectors (see picture)

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

The coordinate system for calculations of components of the vectors $\vec{E}_1$ and $\vec{E}_2$ is shown below. Using vector notation we can write

$$\vec{E}_1 = E_{1x}\hat{i} + E_{1y}\hat{j}, \quad \vec{E}_2 = E_{2x}\hat{i} + E_{2y}\hat{j},$$

then

$$\vec{E}_P = (E_{P})_x\hat{i} + (E_{P})_y\hat{j},$$

and

$$\vec{E}_P = \frac{E_{1x}\hat{i} + E_{1y}\hat{j} + E_{2x}\hat{i} + E_{2y}\hat{j}}{E_1} = \frac{(E_{1x} + E_{2x})\hat{i} + (E_{1y} + E_{2y})\hat{j}}{(E_P)_x^2 + (E_P)_y^2}.$$ 

Thus,

$$(E_{P})_x = (E_{1x} + E_{2x}), \quad (E_{P})_y = (E_{1y} + E_{2y}).$$

First let us note that the charges $q_1$ and $q_2$ are equal $q_1 = q_2 = q$, and are located at the equal distances from the point $P$. Thus $E_1 = E_2 = E$, and

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}, \quad r = \sqrt{b^2 + d^2}.$$ 

It is easy to see form the above picture, that

$$E_{1x} = E \cos \theta_1 = E \cos(2\pi - \theta) = E \cos \theta, \quad E_{1y} = E \sin \theta_1 = E \sin(2\pi - \theta) = -E \sin \theta,$$
\[ E_{x_2} = E \cos \theta_2 = E \cos(\pi + \theta) = -E \cos \theta, \quad E_{y_2} = E \sin \theta_2 = E \sin(\pi + \theta) = -E \sin \theta. \]

Thus,
\[ (E_p)_x = 0, \quad (E_p)_y = -2E \sin \theta, \quad \sin \theta = \frac{d}{r}, \]

or
\[ (\tilde{E}_p)_y = -2 \frac{q}{4\pi \varepsilon_0 \left( b^2 + d^2 \right)} \frac{d}{\sqrt{b^2 + d^2}} = \frac{1}{4\pi \varepsilon_0 \left( b^2 + d^2 \right)^{3/2}} \frac{2qd}{b^2 + d^2}, \]

and
\[ \tilde{E}_p = -\left\{ \frac{1}{4\pi \varepsilon_0 \left( b^2 + d^2 \right)^{3/2}} \frac{2qd}{b^2 + d^2} \right\} \hat{j}. \]

This expression can be simplified further if we take into account that \( b = 2d \). Using this, we get
\[ \tilde{E}_p = -\left\{ \frac{1}{4\pi \varepsilon_0 \left( 4d^2 + d^2 \right)^{3/2}} \frac{2qd}{4d^2 + d^2} \right\} \hat{j} = -\left\{ \frac{1}{4\pi \varepsilon_0 \left( d^2 \right)^{3/2}} \frac{2qd}{d^3 \left( 5 \right)^{3/2}} \right\} \hat{j} = -\left\{ \frac{1}{4\pi \varepsilon_0 \left( d^2 \right)^{3/2}} \frac{2q}{d^3 \left( 5 \right)^{3/2}} \right\} \hat{j}, \]

\[ \tilde{E}_p = -\left\{ \frac{1}{4\pi \varepsilon_0 \left( d^2 \right)^{3/2}} \frac{2q}{5 \sqrt{5}} \right\} \hat{j}. \]

Calculation gives the following
\[ \frac{1}{4\pi \varepsilon_0} \frac{q}{d^2} \frac{2}{5 \sqrt{5}} = \left( 8.99 \cdot 10^9 \frac{N \cdot m^2}{C^2} \right) \left( \frac{10^{-10} C}{m} \right)^2 = \frac{2}{5 \cdot 2.236} = \frac{8.99 \cdot 2}{5 \cdot 2.236} = \frac{10^9}{10^{10}} \frac{N \cdot m^2}{C^2} = 1.608 \cdot 10^6 \frac{N}{C}, \]

Finally,
\[ \tilde{E}_p = -\left( 1.6 \cdot 10^6 \frac{N}{C} \right) \hat{j}. \]

b) The field created at the point \( P \) by the charges \( q_1 \) and \( q_2 \) is oriented in a direction opposite to the direction of the \( y \)-axis. Thus, the charge placed at the point \( D \) has to create the electric field \( \tilde{E}_Q \) at the point \( P \) oriented in the direction parallel to the \( y \)-axis. To have the electric field \( \tilde{E}_p \) compensated we have to demand that
\[ \vec{E}_p + \vec{E}_Q = 0, \quad \Rightarrow \quad \vec{E}_Q = -\vec{E}_p = E_p \hat{j}. \]

The field \( \vec{E}_Q \) is equal (see figure above)

\[ \vec{E}_Q = \left[ \frac{1}{4\pi \varepsilon_0} \frac{|Q|}{h^2} \right] \hat{j}. \]

Now,

\[ E_Q = E_p, \quad \Rightarrow \quad \frac{1}{4\pi \varepsilon_0} \frac{|Q|}{h^2} = E_p, \quad \Rightarrow \quad |Q| = \frac{E_p h^2}{\left(\frac{1}{4\pi \varepsilon_0}\right)}, \]

or

\[ |Q| = \frac{(0.35 m)^2}{8.99 \cdot 10^9 \frac{N \cdot m^2}{C^2}} \left(1.6 \cdot 10^6 \frac{N}{C}\right) = \frac{(0.35)^2}{8.99} \cdot 10^6 \times 1.6 \cdot 10^6 \frac{C^3}{N \cdot m^2} \]

\[ = 0.022 \cdot 10^{-3} C = 2.2 \cdot 10^{-5} C = 22 \cdot 10^{-6} C, \]

\[ Q = -22 \mu C. \]

**II.** Two identical small conducting spheres are placed with their centers 0.5 m apart. One sphere has a charge \( q_1 > 0 \) and another sphere has a charge \( q_2 < 0 \). The value of the force of attraction between these spheres is equal to 7.77 N. The magnitude of the electric field at the middle point \( P \) between two charges is equal to \( 4.32 \cdot 10^6 \frac{N}{C} \). Find (a) the values of the charges \( q_1 \) and \( q_2 \); (b) the value of the electric field at the point \( P \) after the spheres were connected by a conducting wire, electrostatic equilibrium was established and the wire was removed; (c) the electric force between the spheres after wire was removed. The distance between the centers of the spheres remained the same.

**Solution**

(a) The values of the charges \( q_1 \) and \( q_2 \).

The spheres are attracted because the charges have different signs.
\[ |\mathbf{F}_{12}| = |\mathbf{F}_{21}| = F = \frac{1}{4\pi\varepsilon_0} \frac{q_1|q_2|}{d^2}, \quad \Rightarrow \quad q_1|q_2| = F d^2 / \left(\frac{1}{4\pi\varepsilon_0}\right), \]

\[ q_1|q_2| = (7.77 \text{ N})(0.5 \, \text{m})^2 / \left(8.99 \cdot 10^9 \, \text{N} \cdot \text{m}^2 / \text{C}^2\right) = \frac{7.77 \cdot 0.25}{8.99} \cdot 10^{-9} \frac{\mathcal{N} \cdot \mathcal{M}^2 \cdot C^2}{\mathcal{N} \cdot \mathcal{M}^2} = 0.216 \cdot 10^{-9} \text{ C}^2, \]

\[ q_1|q_2| = 216 \cdot 10^{-12} \text{ C}^2. \quad (*) \]

The field at the point P is equal to

\[ \vec{E}_P = \vec{E}_1 + \vec{E}_2 = (E_1 + E_2) \hat{i}, \]

\[ E_1 = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{(d/2)^2}, \quad E_2 = \frac{1}{4\pi\varepsilon_0} \frac{|q_2|}{(d/2)^2}, \quad \Rightarrow \quad E_P = \frac{1}{4\pi\varepsilon_0} \frac{1}{(d/2)^2} (q_1 + |q_2|), \]

\[ (q_1 + |q_2|) = E_P \frac{(d/2)^2}{\left(\frac{1}{4\pi\varepsilon_0}\right)}. \]

\[ (q_1 + |q_2|) = \frac{4.32 \cdot 10^6}{8.99} \frac{\mathcal{M}}{\mathcal{M}} \left(0.25 \mathcal{M}\right)^2 / \left(8.99 \cdot 10^9 \mathcal{M} \cdot \mathcal{M} / \text{C}^2\right) = \frac{4.32 \cdot 0.0625}{8.99} \cdot 10^6 \cdot 10^{-9} \text{ C} = 0.03 \cdot 10^{-3} \text{ C}, \]

\[ (q_1 + |q_2|) = 30.0 \cdot 10^{-6} \text{ C}. \quad (**) \]

Combining equations (*) and (**) we have

\[ q_1|q_2| = 216 \cdot 10^{-12} \text{ C}^2, \quad (q_1 + |q_2|) = 30.0 \cdot 10^{-6} \text{ C}. \]

Introducing \( q_1 = x \cdot 10^{-6} \text{ C} \) and \( q_2 = y \cdot 10^{-6} \text{ C} \), we have

\[ xy \cdot 10^{-12} \text{ C}^2 = 216 \cdot 10^{-12} \text{ C}^2 \]

\[ (x + y) \cdot 10^{-6} \text{ C} = 30.0 \cdot 10^{-6} \text{ C}, \]

or

\[ xy = 216, \quad (x + y) = 30. \]

Solutions of this system of equations are \( x = 12 \) and \( y = 18 \) or vice versa. Thus,

\[ q_1 = 12 \cdot 10^{-6} \text{ C}, \quad \text{and} \quad q_2 = -18 \cdot 10^{-6} \text{ C}. \]
When spheres are connected by a conductor, the redistribution of charge takes place and after the equilibrium is established the spheres have the same charges equal to

\[ q'_1 = q'_2 = q = (q_1 + q_2) / 2, \quad \Rightarrow \quad q = -3 \cdot 10^{-6} \text{ C}. \]

(b) The value of the electric field at the point P the wire was removed.

\[ \vec{E}_p = \vec{E}_1 + \vec{E}_2, \quad \vec{E}_1 = -E_1 \mathbf{\hat{i}}, \quad \vec{E}_2 = E_2 \mathbf{\hat{i}}, \]

\[ q'_1 = q'_2, \quad \Rightarrow \quad E_1 = E_2, \]

\[ \vec{E}_p = 0. \]

(c) The electric force between the spheres after wire was removed

\[ \left| \vec{F}_{12} \right| = \left| \vec{F}_{21} \right| = F' = \frac{1}{4 \pi \varepsilon_0} \frac{|q|^2}{d^2}, \]

\[ F' = \left( 8.99 \cdot 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \left( \frac{3 \cdot 10^{-6} \text{ C}}{0.5 \text{ m}} \right)^2 = \frac{8.99 \cdot 9 \cdot 10^9 \cdot 10^{-12} \text{ N}}{0.25} = 323.6 \cdot 10^{-3} \text{ N} = 0.32 \text{ N}, \]

\[ F' = 0.32 \text{ N}. \]

III. A neutral metal ball is suspended by a string. A positively charged insulating rod is placed near the ball, which is observed to be attracted to the rod. This is because:

A) the ball becomes positively charged by induction
B) the ball becomes negatively charged by induction
C) the number of electrons in the ball is more than the number in the rod
D) the string is not a perfect insulator
E) there is a rearrangement of the electrons in the ball

Answer: E
IV. An electric field intensity depends on:

A) the momentum of a test charge  
B) the kinetic energy of a test charge  
C) the potential energy of a test charge  
D) the force acting on a test charge  
E) the charge carried by a test charge

Answer D

V. Two point charges $q_1$ and $q_2$ are placed a distance $r$ apart. The electric field is zero at a midpoint $P$ between the charges on the line segment connecting them. We conclude that:

A) $q_1$ and $q_2$ must have the same magnitude and sign  
B) $q_1$ and $q_2$ must have the same sign but may have different magnitudes  
C) $q_1$ and $q_2$ must have equal magnitudes and opposite signs  
D) $q_1$ and $q_2$ must have opposite signs and may have different magnitudes

Answer: A

VI. Choose the INCORRECT statement:

A) Gauss’ law states that the net number of lines crossing any closed surface in an outward direction  
is proportional to the net charge enclosed within the surface  
B) Coulomb’s law can be derived from Gauss’ law and symmetry  
C) Gauss’ law applies to a closed surface of any shape  
D) According to Gauss’ law, if a closed surface encloses no charge, then the electric field must  
v Vanish everywhere on the surface

Answer D

VII. When a piece of paper is held with one face perpendicular to a uniform electric field the flux  
through it is $25 \, N \cdot m^2/C$. When the paper is turned $25^\circ$ with respect to the field the flux  
through it is:

A) 0;  
B) $12 \, N \cdot m^2/C$;  
C) $21 \, N \cdot m^2/C$;  
D) $23 \, N \cdot m^2/C$;  
E) $25 \, N \cdot m^2/C$

$$\Phi_E = \vec{E} \cdot \Delta \vec{A} = E \Delta A$$

$$\Phi_E = \vec{E} \cdot \Delta \vec{A} = E \Delta A \cos \theta$$
\[ \Phi_E = E \Delta A \cos \theta = \left( 25 \frac{N \cdot m^2}{C} \right) \cos(25^\circ) = 22.657 \frac{N \cdot m^2}{C} = 23 \frac{N \cdot m^2}{C}. \]

Answer D

VIII. Consider Gauss’s law: \( \oint \vec{E} \cdot d\vec{A} = q / \varepsilon_0. \) Which of the following is true?

A) \( \vec{E} \) must be the electric field due to the enclosed charge
B) If \( q = 0 \), then \( \vec{E} = 0 \) everywhere on the Gaussian surface
C) If the three particles inside have charges of +q, +q, and −2q, then the integral is zero
D) \( \vec{E} \) is everywhere parallel to \( d\vec{A} \) on the surface
E) If a charge is placed outside the surface, then it cannot affect \( \vec{E} \) at any point on the surface

Answer C

IX. A physics instructor in an anteroom charges an electrostatic generator to 25 \( \mu \)C, then carries it into the lecture hall. The net electric flux in \( N \cdot m^2/C \) through the lecture hall walls is:

A) 0
B) 25 \( \cdot 10^{-6} \)
C) 2.2 \( \cdot 10^5 \)
D) 2.8 \( \cdot 10^6 \)
E) can not tell unless the lecture hall dimensions are given

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = q / \varepsilon_0, \quad \Phi_E = \frac{2.5 \cdot 10^{-5} \varepsilon_0}{8.85 \cdot 10^{-12} C^2/(N \cdot m^2)} = 0.282 \cdot 10^7 \frac{N \cdot m^2}{C} = 2.82 \cdot 10^6 \frac{N \cdot m^2}{C}. \]

Answer D