Entering Complex Numbers

You can enter complex numbers in the polar form \( r\angle\theta \), where \( r \) is the magnitude and \( \theta \) is the angle, or polar form \( re^{i\theta} \). You can also enter complex numbers in rectangular form \( a+bi \).

Overview of Complex Numbers

A complex number has real and imaginary components that identify a point in the complex plane. These components are measured along the real and imaginary axes, which are similar to the x and y axes in the real plane.

The point can be expressed in rectangular form or in either of two polar forms.

The \( i \) symbol represents the imaginary number \( \sqrt{-1} \).

As shown below, the form that you can enter depends on the current Angle mode.

<table>
<thead>
<tr>
<th>You can use the form:</th>
<th>When the Angle mode setting is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a+bi )</td>
<td>Radian or Degree</td>
</tr>
<tr>
<td>( re^{i\theta} )</td>
<td>Radian only</td>
</tr>
<tr>
<td>( (r\angle\theta) )</td>
<td>(In Degree angle mode, this form causes a Domain error.)</td>
</tr>
</tbody>
</table>

Use the following methods to enter a complex number.

<table>
<thead>
<tr>
<th>To enter the:</th>
<th>Do this:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular form ( a+bi )</td>
<td>Substitute the applicable values or variable names for ( a ) and ( b ).</td>
</tr>
<tr>
<td></td>
<td>( a \text{ [enter]} b \text{ [enter]} [i] )</td>
</tr>
<tr>
<td>For example:</td>
<td>\begin{align*} 2+3\cdot i &amp; \quad 2+3\cdot i \ 2\cdot 5#i &amp; \quad 2\cdot 5#i \end{align*}</td>
</tr>
</tbody>
</table>

Note: To get the \( i \) symbol, press [2nd] [i]. Do not simply type an alphabetic \( i \).
To enter the:  

Polar form $re^{i\theta}$  

- or -  

$(r, \theta)$  

Do this:  

Substitute the applicable values or variable names for $r$ and $\theta$, where $\theta$ is interpreted according to the Angle mode setting.  

**TI-89:**  

- or -  

**TI-92 Plus:**  

- or -  

For example:

<table>
<thead>
<tr>
<th>i</th>
<th>$\sqrt{2}$</th>
<th>$\sqrt{2}$ + $2 \cdot i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \cdot e^{\frac{\pi}{4}}$</td>
<td>$2 + 2 \cdot i$</td>
<td></td>
</tr>
<tr>
<td>$(2 + \frac{\pi}{4})$</td>
<td>Result is shown in rectangular form, but you can select polar form.</td>
<td></td>
</tr>
</tbody>
</table>

**Complex Format Mode for Displaying Results**

Use the MODE to set the Complex Format mode to one of three settings.  

You can enter a complex number at any time, regardless of the Complex Format mode setting. However, the mode setting determines how results are displayed.

If Complex Format is:  

**The TI-89 / TI-92 Plus:**

- **REAL**  
  
  Will not display complex results unless you:
  
  - **Enter a complex number.**  
  - **Use a complex function such as cFactor(), cSolve(), or cZeros().**
  
  If complex results are displayed, they will be shown in either $a+bi$ or $re^{i\theta}$ form.

- **RECTANGULAR**  
  
  Displays complex results as $a+bi$.

- **POLAR**  
  
  Displays complex results as:
  
  - $re^{i\theta}$ if the Angle mode = Radian
  
  - $(r, \theta)$ if the Angle mode = Degree
Using Complex Variables in Symbolic Calculations

Regardless of the Complex Format mode setting, undefined variables are treated as real numbers. To perform complex symbolic analysis, you can use either of the following methods to set up a complex variable.

Method 1: Use an underscore (TI-89: [\_] TI-92 Plus: [\_\_]) as the last character in the variable name to designate a complex variable. For example:

\[ z_\_ \text{ is treated as a complex variable} \]

\[ \text{unless } z \text{ already exists, in which case it retains its existing data type}. \]

Method 2: Define a complex variable. For example:

\[ x+y\_i \]

Then \( z \) is treated as a complex variable.

Complex Numbers and Degree Mode

Note: If you use Degree angle mode, you must make polar entries in the form \((r, \theta)\). In Degree angle mode, an \( r \) entry causes an error.

Radian angle mode is recommended for complex number calculations. Internally, the TI-89/TI-92 Plus converts all entered trig values to radians, but it does not convert values for exponential, logarithmic, or hyperbolic functions.

In Degree angle mode, complex identities such as \( e^\theta \cos(\theta) = \cos(\theta) + i \sin(\theta) \) are not generally true because the values for \( \cos \) and \( \sin \) are converted to radians, while those for \( e^\theta \) are not. For example, \( e^{i(45)} = \cos(45) + i \sin(45) \) is treated internally as \( e^{i(45)} = \cos(\pi/4) + i \sin(\pi/4) \). Complex identities are always true in Radian angle mode.
Using Complex Numbers

A complex number has two components: real (a) and imaginary (+bi). On the TI-86, you enter the complex number a+bi as:

- \((real, imaginary)\) in rectangular form
- \((magnitude, angle)\) in polar form

You can enter a complex number in rectangular or polar form, regardless of the current complex number mode setting. The separator (, or \(\pm\)) determines the form.

- To enter rectangular form, separate real and imaginary with a comma (,).
- To enter polar form, separate magnitude and angle with an angle symbol (\(\angle\)).

Each component \((real, imaginary, magnitude, or angle)\) can be a real number or an expression that evaluates to a real number; expressions are evaluated when you press [ENTER].

When RectG complex number mode is set, complex numbers are displayed in rectangular form, regardless of the form in which you enter them (as shown to the right).

When PolarG complex number mode is set, complex numbers are displayed in polar form, regardless of the form in which you enter them (as shown to the right).

Complex Results

Complex numbers in results, including list, matrix, and vector elements, are displayed in the form (rectangular or polar) specified by the mode setting (Chapter 1) or by a display conversion instruction (page 61).

- When Radian angle mode is set, results are displayed as (magnitude,angle).
- When Degree angle mode is set, results are displayed as (real,imaginary).

For example, when PolarG and Degree modes are set, \((2,1)-(1,45)\) returns \((1.32565429614/12.764398628)\).

Using a Complex Number in an Expression

- Enter the complex number directly.
- Use the ALPHA keys, alpha keys, and other character keys to enter a complex variable.
- Select a complex variable from the VARS CPLX screen.

The CPLX (Complex Number) Menu [F3] [CPLX]

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>conj</td>
<td>real</td>
</tr>
<tr>
<td>conj</td>
<td>real,imaginary</td>
</tr>
<tr>
<td>conj</td>
<td>magnitude,angle</td>
</tr>
<tr>
<td>real</td>
<td>real,imaginary</td>
</tr>
<tr>
<td>real</td>
<td>magnitude,angle</td>
</tr>
<tr>
<td>imag</td>
<td>real,imaginary</td>
</tr>
<tr>
<td>imag</td>
<td>magnitude,angle</td>
</tr>
<tr>
<td>abs</td>
<td>real,imaginary</td>
</tr>
<tr>
<td>abs</td>
<td>magnitude,angle</td>
</tr>
<tr>
<td>Command</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td><code>angle (real, imaginary)</code></td>
<td>Returns the polar angle of a complex number, list, vector, or matrix calculated as ( \tan^{-1} \frac{\text{imaginary}}{\text{real}} ) (adjusted by ( \pi ) in the second quadrant or ( -\pi ) in the third quadrant); the result is ( \tan^{-1} \frac{\text{imaginary}}{\text{real}} )</td>
</tr>
<tr>
<td><code>angle (magnitude, angle)</code></td>
<td>Returns angle (where ( \text{angle} := \text{radians} ))</td>
</tr>
<tr>
<td><code>complexValueRec</code></td>
<td>Displays <code>complexValue</code> in rectangular format ((\text{real}, \text{imaginary})), regardless of complex mode setting; valid only at the end of a command and only when <code>complexValue</code> is indeed complex</td>
</tr>
<tr>
<td><code>complexValuePol</code></td>
<td>Displays <code>complexValue</code> in polar format ((\text{magnitude}, \text{angle})), regardless of complex mode setting; valid only at the end of a command and only when <code>complexValue</code> is indeed complex</td>
</tr>
</tbody>
</table>

Select \( \{ \text{and} \} \) from the LIST menu.

You must enter commas to separate list elements.

You can enter a complex list, vector, or matrix directly. The syntax below is for lists. To enter a complex vector or matrix, substitute brackets for braces below and use the correct form for either data type (Chapters 12 and 13).

In rectangular form, to use lists of complex numbers with `conjugate`, `real`, `imag`, `abs`, and `angle`, the syntax is:

\[
\text{conjugate}[(\text{real}_A, \text{imag}_A), (\text{real}_B, \text{imag}_B), (\text{real}_C, \text{imag}_C), \ldots]
\]

In polar form, to use lists of complex numbers with `conjugate`, `real`, `imag`, `abs`, and `angle`, the syntax is:

\[
\text{real}[(\text{magnitude}_A, \text{angle}_A), (\text{magnitude}_B, \text{angle}_B), (\text{magnitude}_C, \text{angle}_C), \ldots]
\]

When you use a list the TI-86 calculates the result element by element and returns a list, in which each element is expressed according to the complex mode setting.