Problem 1

Given:
\[ G(s) = \frac{10}{(s+2)(s+5)} \quad H(s) = 1 \]

1. Find the steady state error to a unit step input with the proportion gain given (B).
2. Design a lag controller to reduce the amount of error by a factor of 10 (or \( e_{ss} \) original times 0.1)

1) Unity Feedback Type 0 System

\[ e_{ss} = \frac{1}{1+k_p} \quad k_p = \lim_{s \to 0} \frac{G(s)}{G(s)} = \frac{10}{(0)(5)} = 1 \]

\[ e_{ss} = \frac{1}{1+1} = 0.5 \]

2) Want \( e_{ss} = 0.05 \)

\[ e_{ss} = 0.05 = \frac{1}{1+k_p} \Rightarrow 0.05(1+k_p) = 1 \]

\[ 0.05 + 0.05k_p = 1 \]

\[ 0.05k_p = 0.95 \]

\[ k_p = 19 \]

Now

\[ \Rightarrow \text{Need to improve } k_p \text{ by Factor of 19} \]

\[ G_{lag} = \frac{s + 0.019}{s + 0.001} \]

Note: There are many solutions.

This is just one.
Problem 2

Given:

\[ G(s) = \frac{1}{(s+2)(s+5)}, H(s) = 1 \]

Design a PD controller such that the damping ratio of the dominant second order poles is 0.707.

* First Note: if \( \xi = 0.707 \) is the only requirement, a proportional controller would do the job.

\[ \text{CL Poles } @ -3.5 \pm 3.5j \]

\[ k = \frac{1.5^2 + 3.5^2}{1.5^2 + 3.5^2} \]

\[ k = 14.5 \]

Assume \( y \) is to the left of -5

Pick \( z_c = +15 \)

\[ \Theta_c = \tan^{-1}\left( \frac{y}{15-y} \right) \]

\[ \Theta_p = 180 - \tan^{-1}\left( \frac{y}{y-5} \right) \]

\[ \Theta_d = 180 - \tan^{-1}\left( \frac{y}{y-2} \right) \]

Using \( \Theta_d < \Theta_p : 180 - \Theta_c - \Theta_p - \Theta_d \)

Solve for \( y = 10.3 \)

\[ k = \frac{\sqrt{10.3^2(10.3-5)^2}}{\sqrt{10.3^2 + (10.3-2)^2}} \]

\[ k = 13.5 \]