LEAD COMPENSATION

\[ G_C = \frac{K(s + b)}{(s + P_c)} \]

- **Not a pure differentiator**
- **Advantages:** Do not need active components.
- **Not differentiating so not amplifying high freq. noise.**

**Same idea w/ design as PD controller**

- **Choose a desired CL pole**
- **Choose a compensator zero location**
  - May want to look at P.L. to see effect.
- **Solve for the compensator pole location using a criterion.**
- **Solve for gain using magnitude criterion.**
- **Check w/ simulations to see if approximations are ok.**

**Ex:** Same TP. \( G_p = \frac{1}{s(5s + 10)} \) as PD design w/

- **Same desired CL poles:** \(-3 + j3\)
  - \( T_3 = 1.5 \) sec.

- **Select zero of compensator \( z = -2 \)**

\( \theta_0 = \theta_{pd} \cdot \left[ \theta_p1 + \theta_p2 + \theta_p3 + \theta_{pc} \right] \)

**Solve for \( \theta_{pc} \)**

\( \theta_{pc} = 180 - \tan^{-1}(\frac{3}{2}) = 108.43^\circ \)

\( \theta_p1 = 180 - \tan^{-1}(\frac{3}{4}) = 135^\circ \)

\( \theta_p2 = \tan^{-1}(\frac{1}{4}) = 71.57^\circ \)

\( \theta_p3 = \tan^{-1}(\frac{3}{6}) + \tan^{-1}(\frac{1}{3}) = 45^\circ \)
\[ 180 = 108.4 - [135 + 91.5 + 45 + \Theta_{pc}] \]
\[ 180 = -143 - \Theta_{pc} \]
\[ \Theta_{pc} = 36.87° \]

\[ \tan(36.87°) = \frac{3}{p_c - 3} \]
\[ 0.75(p_c - 3) = 3 \]
\[ p_c = \frac{5.25}{0.75} \]
\[ p_c = 7 \]
\[ C_0 = \frac{k(5+2)}{(5+7)} \]

Find \( k = \frac{\pi/p_c}{\pi/4} \)
\[ k = 90 \]
\[ C_0 = \frac{90(5+2)}{5+7} \]

Look at what we really have

CL poles: \(-3.3, -1\)
\(-10 \quad \text{YIKES! SLOWER 1st ORDER POLE} \)

Ce zero: \(-2\)

\[ T_c = 3.63 \]
\[ OS = 0.70 \]
CONTROLS

SELECT \( z_c = -4 \) (will get P/I cancellation)
\( z_c \) NEVER do with unstable P/I

\[ +180 = \Theta_p - \Theta_1 - \Theta_3 - \Theta_{pc} \]
\[ +180 = [1 + \tan(\frac{\Theta}{2})] - \tan(\frac{\Theta}{2}) \cdot \Theta_{pc} \]
\[ \Theta_{pc} = 0^\circ \quad \text{WHAT DOES THIS MEAN?} \]
How would we get an angle of ZER0?

\[ P_c = \infty \quad k = \infty \quad \text{IS THIS POSSIBLE?} \]

Choose \( P_c = 10000 \)
\[ K = 180000 \]

\( M = 1.5 \)
\( \% \text{OSS} = 4.7 \%

MEETS SPECS, BUT IS IT OBTAINABLE \( G_c = \frac{180000(1+y)}{s + 10000} \)

CL poles:
- \( -\frac{1}{3} \pm j \frac{2}{3} \)
- \( -10000 \)

CL zero:
- \( -y \)
Select $z_e = -5$

\[\pm 180 = \theta_e - [\theta_p + \theta_p2 + \theta_p3 + \theta_{pe}] \]
\[\pm 180 = \text{tan}(\frac{\pi}{2}) \cdot [15\pi + 7.5\pi + 45 + \theta_{pe}] \]
\[\pm 180 = -195.25 - \theta_{pe} \]
\[\theta_{pe} = -15.25 \]

How would we get this?

As $\theta_e = \infty$, $\theta = 180^\circ$
As $\theta_e = -\infty$, $\theta = 0$

But we need a new angle $\frac{\pi}{4} - 15.25^\circ$

In this case $p_e$ would have to be complex, but it doesn't have a conjugate because we are only adding 1 pole.

Thus cannot have $z_e = -5$

Select $z_e = 3$

\[\pm 180 = \theta_e - [\theta_p + \theta_p2 + \theta_p3 + \theta_{pe}] \]
\[\pm 180 = 90 \cdot [15\pi + 7.5\pi + 45 + \theta_{pe}] \]
\[\pm 180 = -161.5\pi - \theta_{pe} \]
\[\theta_{pe} = 11.44 \]
\[\text{tan}(18.44) \approx \frac{3}{2} \]
\[0.733 (7e - 3) + 3 \]
\[\theta = 12 \]
\[K = \frac{13^2 - 3^2}{12^2 - 3^2} \cdot \frac{5^2 - 3^2}{12^2 + 3^2} \cdot \frac{6^2 + 3^2}{12^2 + 3^2} \]
\[K = 180 \]

\[G_e = \frac{180}{(s + 3)^2} \]

CP poles: -13.83, -3.33, -216
CC zero: -3

Note: A little faster (due to effects of NO 08)
Controls.

Improving S.S. Error & Transient Response:

\[ G_c = \frac{k(s^2 + k_2)k_1}{s} \rightarrow \text{2 zeros, 1 pole @ origin} \]

\[ G_c = \frac{k(s^2 + k_2)}{s} \]

- 1 zero used for ideal integral compensator (close to origin (near pole))
- 1 zero used for ideal derivative

Design Strategy:

- Evaluate the performance of the uncompensated system.
  → Determine what improvements you would like to make.

- Design a P-I controller (gain & zero location) to achieve these goals
  → Simulate to check performance & control effort
  → Iterate if necessary

- Design a PI controller to meet S.S. error goal
  → Simulate to check
  → If implementing w/ k_1, k_p, k_i back these gains out.

* Note Pd improper so it has the same problems with amplifying noise verses a PI controller.
LEAD/LAG CONTROLLER

\[ C(s) = \frac{K(s + P_{lag})(s + P_{lead})}{(s + P_{lag})(s + P_{lead})} \]

**Design Strategy**

1. **Evaluate Sys. Performance (Uncompensator)**
   - Determine Controller goals
2. **Design a Lead Compensator (Gain, P_{lag}, P_{lead})**
   - Simulate to check performance & control effort (\(\zeta_s\))
   - Evaluate \(\zeta_s\) & set goal for \(\zeta_s\)
3. **Design a Lag Compensator to Achieve the Goal**
   - Simulate to verify goals are met.
**CONTROLS**

**EX** 
**Given** \( G(s) = \frac{e^{-5s}}{5s+5(s+11)} \) \( H(s) = 1 \)

**Find:** \( K \) to operate \( w/30\% \) os for the uncomp. sys.
\( T_p \) & \( K_v \) for uncomp. sys.

**Design a lead/lag compensator to decrease** \( T_p \) **by a factor of 2**, decrease \( 70\% \) os **by a factor of 2** & **improve** \( \varepsilon_{ss} \) **by a factor of 30**

\( 30\% \) os \( \rightarrow 0.3579 + j \)

**Using MATLAB**

\( k = 217, \%05 = 30\% \)

(See Matlab Plot)

Cl poles & \( k = 217 \): \(-1.4 e^{5.8} \)
\(-13.05 \)

So 2nd order approx. unwin.

\( T_p = \frac{\pi}{\omega_n} = 0.8267 \ sec \) (uncomp)

\( k_p = \lim_{s \to 0} s G(s) = \frac{217}{(6)(11)} = 3.94 \)

\( \omega_{ss} = \frac{
}{\text{uncomp}} \)

So: Goals of controller:
\( \%05 < 15\% \)
\( T_p < 0.413 \ sec \)
\( K_v = 3.94(10) = 39.4 \) \( \rightarrow \) Lead

**Translate 70% & Tp to a desired cl pole**

\( 70\% \) \( \rightarrow s = 0.517 \)

\( \theta = \cos^{-1}(0.517) = 68.9 \)
\( \phi = \sin^{-1}(0.517) = 31.13 \)

\( T_p = \frac{\pi}{\omega_n} \rightarrow \omega_n = \frac{\pi}{T_p} = \frac{\pi}{7.6} = 7.6 \)

\( \tan(68.9) = \frac{7.6}{\phi} \)
\( \phi = 2\pi \tan(58.9) = 4.59 \)

**Desired cl pole** \(-4.59 + 7.6j\)
**Choose** \( \theta_{cemo} = -5 \)

\[
\begin{align*}
\theta_{cemo} &= \theta_2 + \theta_3 + \theta_4
\end{align*}
\]

\[
\tan^{-1} \left( \frac{20}{90} \right) - \tan^{-1} \left( \frac{25}{50} \right) + \tan^{-1} \left( \frac{20}{90} \right) + \tan^{-1} \left( \frac{25}{50} \right)
\]

\[
\frac{1800}{150} = 54.54 - 63.46 - \theta_{cemo}
\]

\[
\theta_{cemo} = 9.02^\circ
\]

\[
\tan(90.2) = \frac{26}{PC - 4.59}
\]

\[
0.159(\text{PC} - 4.59) = 7.4
\]

\[
0.159\text{PC} - 0.73 = 7.4
\]

\[
\text{PC} = 52.5
\]

**Kemno**

\[
\begin{align*}
\frac{1.65}{52.5} \times \frac{7.6^2}{\sqrt{(11.44)^2 + 7.6^2}}
\end{align*}
\]

\[
\frac{1.65}{52.5} \times \frac{7.6^2}{\sqrt{(11.44)^2 + 7.6^2}}
\]

\[
(41.5) \times (9.44)(4.88)
\]

\[
K = \frac{4282.00}{2022.00}
\]
LAG COMPENSATOR

Required $K_0 = 118.2$

W/ LOAD Comp. $K_p = \frac{(42\times2.5)}{(5\times11.52.5)} = 7.41$

So WE NEED $\frac{118.2}{7.41} = 15.95 \rightarrow \frac{\theta_c}{\theta_c > \theta_c}$ by 15.95

$\frac{\theta_c}{\theta_c} = 15.75$

$G_{lag} = \frac{5 + 0.01595}{(s + 0.001)}$

$G_c = \frac{42\times2.5(s + 5)(s + 0.01595)}{(s + 52.5)(s + 0.001)}$
Controls Spot Ziegler Nichols

ZIEGLER-NICHOLS TUNING OF PID REGULATORS (4.4.2)

- In order to design our PID controllers, what have we known
  GIVEN: \( G_p = \) __________

How do we get \( G_p \)?

→ Modelling from 1st principles
  → Can take a lot of time = $$$
  → May change a/process LLDPE vs. LDPE

→ Testing
  → Needs to cover all op. conditions

In industry most controllers are PID

- Can we find acceptable values for \( K_i, K_p, K_d \) without knowing \( G_p \)?

Ziegler and Nichols (1942, 1943) noticed that many processes exhibit a "Process Reaction Curve" to a step input.

\[
y(t) = \frac{A}{t} = \text{Reaction Rate}
\]

\( A \)

\( t \)

\( L + t_d \)
This curve is characteristic of many systems and can be approximated by \[ \frac{V(s)}{R(s)} = \frac{Ae^{-\frac{ts}{\tau}}}{\tau s + 1} \]

→ **First order system, w/ time delay, \( t_d \)**

→ \( e^{-\frac{ts}{\tau}} \) → See Note: Doesn't change magnitude, pure phase lag.

For approximation can be used to approximate w \( \frac{S^{t_a}}{S^{t_b}} \) terms

**Two methods for tuning**

1) \( \frac{1}{4} \) decay ratio

**Transient delay** to \( \frac{1}{4} \) of its value in 1 period

\[ G_c(s) = K_p \left( 1 + \frac{1}{\tau_i s} + T_d s \right) \]

\[ \rightarrow \quad K_p \left( 1 + \frac{K_p}{T_i} \left( \frac{1}{T_i} + \frac{T_d}{K_p} \right) \right) \]

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>Optimal Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional</td>
<td>( K_p = \frac{1}{RL} )</td>
</tr>
<tr>
<td>( K_p = 0.9/RL )</td>
<td>( T_i = \frac{1}{0.3} )</td>
</tr>
<tr>
<td>PI</td>
<td>( K_p = 1.2/RL )</td>
</tr>
<tr>
<td>( T_i = \frac{1}{L} )</td>
<td>( T_d = 0.5L )</td>
</tr>
</tbody>
</table>

\( R = \frac{A}{L} \) = Reaction Rate

\( L = t_d \)
2) ULTIMATE SENSITIVITY METHOD

→ BASED ON EVALUATING THE AMPLITUDE & FREQUENCY OF OSCILLATIONS OF THE SYSTEM AT THE LIMIT OF STABILITY RATHER THAN ON TAKING STEP RESPONSES.

→ INCREASE K UNTIL MARGINALLY STABLE (AMPLITUDE LIMITED BY ACTUATOR SATURATION)
   → GAIN HERE IS K_u (ULTIMATE GAIN)
   → PERIOD OF OSCILLATIONS = P_u (ULTIMATE PERIOD)

\[ G_c = k_u \left( 1 + \frac{1}{\frac{P_u}{2}} + \frac{T_{PD}}{T_u} \right) \]

- **PID: **
  \[ K_p = 0.5 K_u \]
  \[ T_i = \frac{P_u}{1.2} \]
  \[ T_d = \frac{0.4 P_u}{P_u} \]
  \[ T_{PD} = \frac{1}{5} P_u \]

→ SOME PLC'S COME WITH "AUTOMATIC TUNING".

→ USUALLY A SYSTEM IS GIVEN A PERTURBATION (SMALL STEP RESPONSE) & SYSTEM IS ADJUSTED IN THAT A 1/4 DELAY METHOD.