Problem 1

Given a unity feedback system, use frequency response methods to determine the value of gain, \( K \), to yield a step response with a 20% overshoot. \( G(s) = \frac{K}{s(s^2+6s+12)} \)

\[
S = \frac{-\ln(20/100)}{\sqrt{s^2+6s+12}} = \frac{-\ln(20/100)}{\sqrt{s^2+6s+12}}
\]

\( S = 0.4559 \)

\[
\theta_m = \tan^{-1}\left(\frac{2S}{\sqrt{S^2+4.5^2}}\right) \cdot \tan^{-1}\left(\frac{2(0.4559)}{\sqrt{2(0.4559)^2+4.5^2}}\right)
\]

\( \theta_m = 48.15^\circ \)

So, at the gain crossover frequency (where \( \theta_m = 0 \)), the phase should be

\[-180 + 48.15 = -131.85^\circ \]

From uncompensated Bode (see plot)

\( \omega = 3 \) @ phase \(-132^\circ \)

Magnitude = -48.3 dB

We need to make this 0.18 by increasing gain by \( K = 10 \)

\( K = 260 \)
Bode for Problem 1
Problem 2

Given a unity feedback system with \( G(s) = \frac{k}{s(s+1)} \)

The system is operating with 15% overshoot.

Find a compensator to improve the steady state error constant to \( K_v = 50 \) without changing the 10% using a lag controller.

For a \( K_v = 50 \)

Magnitude @ 0.5 rad/sec = 40 dB

With \( k = 1 \) \( M_v \) at \( \omega = 0.5 \text{ rad/sec} = -10.948 \)

We need to add \( 40 \text{ dB} + 10.948 = 50.948 \text{ dB} \)

\[ K = 10^{\left(\frac{50.948}{20}\right)} = 356.75 \]

For 15% 0.5 \( \rightarrow \) \( \phi = 0.517 \rightarrow \) \( \phi_m = 53.17^\circ \)

\( \phi_m \) desired = 53.17 + 10 = 63.17

Need to find \( \phi_m \) where phase = -180 + 63.17 = -116.8

See plot \( \rightarrow \) \( \omega_m = 3.51 \text{ rad/sec} \)

@ \( \omega = 3.51 \text{ rad/sec} \) \( M_v \) = 22.1 dB

So we need to add 22.1 dB of attenuation at this frequency with lag compensator

\[ \omega_s = 10 \omega_m = 0.351 \lesssim \]

\[ \omega_c = \omega_s \times 10 = 22.1 \times 10 = 0.351 \times 10 = 2.75 \times 10^{-2} \]

\[ G_c(s) = \frac{356.75 \cdot 2}{s(0.351 + 1)} \]

\[ s + 0.00075 \]
See plot of compensated sys.

Actual $P_m = -122 - (-170) = 58^\circ$

Actual $\% OS = 14.4\% < 0.15$. 
Problem 2 for gain adjustment

Problem 2 for Lag Calculation
Problem 2 - Step Response for Compensated
Problem 3

Given a unity feedback system with \( G(s) = \frac{K}{s^2 + 5s + 7} \)

This system is operating with 15% OS. Find a compensator to have a 5-fold improvement in steady state error without changing the overshoot with a PI controller.

\[ \Rightarrow \quad \text{Type 0 System} \quad \epsilon_s = \frac{1}{1 + K_p} \]

For 70.05% \( \Rightarrow \) \( 5 = 0.8169 \Rightarrow P_m = 53.17 \)

Look where phase \(-780 + 53.17 = -126.8^\circ\)

This happens @ \( w = 3.75 \) rad/sec

\[ \text{Mag} @ 3.75 = -46.5 \, dB \]

So \( K = 10 \frac{46.5}{20} = 211.35 \quad \Rightarrow \quad \text{Simulations yields 53\% OS} \]

Using root locus \( K = 97.7 \) for 2nd order system \( \bar{s} = 0.517 \)

Yields \( 70.05 \approx 13.3 \)

\( K = 106.5 \Rightarrow 157 \). Through simulation

From Bode Plot \( K = 106.5 \)

Low freq. gain = 364

\[ \log_{10}(Kp) = 3.64 \quad 3.64 = 20 \log_{10}(Kp) \]

\[ K_p = 10 \quad 1.52 \]

Desired \( \epsilon_s = \frac{0.397}{5} = 0.0793 \)

\[ 0.0793 = \frac{1}{1 + K_p} \quad \Rightarrow \quad K_p = 11.6 \]

Low freq. gain in dB = 20 \log_{10}(11.6) = 21.3 \, dB <-

Need to raise mag. Bode plot by \( K = 10 \)

\[ \frac{(21.3 - 3.62)}{20} = K = 7.65 \]
Problem 3 cont.

Total gain before lag element = 7.65 + 106.5 = 814.7

For 15% OS need \( P_m = 53.17 \times 10 \) = 63°

\[ P_m = 53.17 \times 10 = 63° \]

Next, we need to find where 63° \( P_m \) happens.

\[ 63° - 180° = -117° \rightarrow \text{Find where this freq happens} \]

From book using \( K = 814.7 \)

\[ w = 3.3 \text{ rad/sec} \quad \phi = -117° \text{ phase} \]

\[ \text{Mag} @ 3.3 \text{ rad/sec} = 13.1 \text{ dB} \]

So, the lag controller needs to reduce \( M_{ss} \) by 13.1 dB @ 3.3 rad/sec.

\[ w_2 = \frac{1}{10} w_{nm} = \frac{1}{10} (3.3) = 0.33 \]

\[ w = \frac{w_2}{10} \quad \frac{w_2}{10} = \frac{3.3}{10} = 0.33 \]

\[ w = 0.33 \]

\[ G_c(s) = \frac{814.7 (\frac{s}{0.33} + 1)}{5 (\frac{s}{0.0729} + 1)} \]

From simulation, \( e_{ss} = 1.0121 \times 0.079 \)

\( \% \) OS = 87°
Problem 3 – after adjusting for %os=15%

Problem 3 – for lag compensator calc.
Problem 3 – Time Response after compensation