18. Assume a particular solution of

Substitute into the differential equation and obtain:

\((7C + 2D) \cos(2\ell) + (-2C + 7D) \sin(2\ell) = 5 \cos(2\ell)\)

Equating like coefficients,

\(7C + 2D = 5\)

\(-2C + 7D = 0\)

From which, \(C = \frac{35}{53}\) and \(D = \frac{10}{53}\).

The characteristic polynomial is

\(\lambda^2 + 7 = 0\)

Thus, the total solution is

\[x(t) = A e^{-7\ell} \left(\frac{35}{53} \cos(2\ell) + \frac{10}{53} \sin(2\ell)\right)\]

Solving for the arbitrary constants, \(x(0) = A \frac{35}{53} = 0\). Therefore, \(A = \frac{35}{53}\). The final solution is

\[x(t) = \left(-\frac{35}{53}, e^{-7\ell} \left(\frac{35}{53} \cos(2\ell) + \frac{10}{53} \sin(2\ell)\right)\]

19. Assume a particular solution of

\[x_0(\ell) = C \cos(2\ell) + D \sin(2\ell)\]

Substitute into the differential equation and obtain

\[-2(C - 2D) \cos(2\ell) - 4(C + \frac{1}{2}D) \sin(2\ell) = \sin(2\ell)\]

Equating like coefficients,

\[-2(C - 2D) = 0\]

\[-4(C + \frac{1}{2}D) = 1\]

From which, \(C = -\frac{1}{2}\) and \(D = -\frac{1}{10}\).

The characteristic polynomial is

\(\lambda^2 + 2\lambda + 2 = (\lambda + i) (\lambda + 1 - i)\)

Thus, the total solution is

\[x = -\frac{1}{6} \cos(2\ell) - \frac{1}{10} \sin(2\ell) + e^{-\ell} \left(4 \cos(\ell) + 2 \sin(\ell)\right)\]

Solving for the arbitrary constants, \(x(0) = A - \frac{1}{2} = 1\). Therefore, \(A = \frac{11}{5}\). Also, the derivative of the solution is

\[\frac{dx}{dt} = -\frac{1}{3} \cos(2\ell) + \frac{2}{3} \sin(2\ell) + (-A + B) e^{-\ell} \cos(\ell) - (A + B) e^{-\ell} \sin(\ell)\]

Solving for the arbitrary constants, \(\dot{x}(0) = -A + B - 0.2 = -3\). Therefore, \(B = -\frac{3}{5}\). The final solution is

\[x(t) = -\frac{1}{5} \cos(2\ell) - \frac{1}{10} \sin(2\ell) + e^{-t} \left(\frac{11}{5} \cos(\ell) - \frac{3}{5} \sin(\ell)\right)\]