Consider the unit step response of a unity feedback control system whose open loop transfer function is \( G(s) = \frac{G}{s(s+1)} \)

\[ T(s) = \frac{G}{1+GH}; \quad H(s) = 1 \]

\[ T(s) = \frac{s(s+1)}{s(s+1)} \]

Find: \( T_r, T_p, T_\infty, T_s \)

**General 2nd Order TF:** \( \frac{w_n^2}{s^2 + 2Sw_n + w_n^2} \)

\( w_n^2 = 1 \Rightarrow w_n = 1 \leq \)

\( \frac{1}{2Sw_n^2} = 1 \)

\( S = \frac{1}{2w_n^2} = \frac{1}{2(1)} \)

\( S = 0.5 \leq \)

\( \frac{1}{S^2 + 0.5} \)

\[ T_p = \frac{\pi}{\sqrt{1 - 0.5^2}} \]

\[ T_p = \frac{\pi}{\sqrt{1 - 0.5^2}} \]

\[ T_p = 3.63 \text{ sec} \]

\( \%\text{O.S.} = e^{-\frac{\pi T}{\sqrt{1 - 0.5^2}}} \times 100 \)

\( \%\text{O.S.} = 16.37 \% \)

\[ T_s = \frac{4}{5w_n} = \frac{4}{0.5} \]

\[ T_s = 8 \text{ sec} \]

\[ T_r = \frac{\pi - \beta}{\omega_n} \]

\( \beta = \tan^{-1}\left(\frac{\omega_n}{\beta}\right) \)

\( \beta = \tan^{-1}\left(\frac{\omega_n}{\frac{\omega_n}{0.5}}\right) = \tan^{-1}\left(\frac{\frac{S^2}{\omega_n}}{0.5}\right) = 1.047 \)

\[ T_r = \frac{\pi - 1.047}{(0.5)\sqrt{0.5}} = 2.418 \text{ sec} \]

\[ T_s = 2.418 \text{ sec} \]
Problem B-5-3

Consider the closed-loop system given by

\[ C(s) = \frac{\omega_n^2}{\frac{1}{100}} = \frac{\omega_n^2}{s^2 + 25\omega_n s + \omega_n^2} \]

Determine values of \( S \) and \( \omega_n \) so that the system responds to a step input with approximately 5% overshoot and with a settling time of 2 sec (27s).

\[ S = \frac{-\ln \left( \frac{7005}{100} \right)}{\sqrt{\pi^2 + \ln^2 \left( \frac{7005}{100} \right)}} \]

\[ S = \frac{-\ln \left( \frac{5}{100} \right)}{\sqrt{\pi^2 + \ln^2 \left( \frac{5}{100} \right)}} = \]

\[ S = 0.69 \leq \]

\[ T_s = \frac{4}{5\omega_n} \]

\[ \omega_n = \frac{4}{T_s} = \frac{4}{0.69} \]

\[ \omega_n = 2.898 \text{ rad/sec} \]
Problem 8-5-4

The figure is a block diagram of a space-vehicle attitude-control system. Assuming the time constant $T$ of the controller to be 3 sec, and the ratio $\frac{K}{s}$ to be $\frac{3}{4}$ rad/sec$^2$.

Find the damping ratio of the system.

\[ T(s) = \frac{K(Ts+1)}{Js^2} = \frac{K(Ts+1)}{s^2 + KTs + K} \]

\[ = \frac{\frac{K}{s}(Ts+1)}{s^2 + [\frac{K}{2}]s + \frac{K}{4}} = \frac{\frac{K}{s} T(s + \frac{1}{T})}{s^2 + \frac{K}{2}s + \frac{K}{4}} \]

\[ \omega_n^2 = \frac{K}{2} = \frac{3}{4} \]

\[ \omega_n = \frac{\sqrt{3}}{2} \]

\[ \omega_h = 0.4714 \text{ rad/sec} \]

\[ 2\omega_h = \frac{\omega_n}{2} = \frac{\sqrt{3}}{4} \]

\[ 2\omega_h = \frac{\omega_n}{2} = \frac{\sqrt{3}}{4} \]

\[ S = \frac{2\omega_h}{2\omega_h} = \frac{2}{2\sqrt{3}} = \frac{2}{2\sqrt{3}} \]

\[ S = \frac{2}{2\sqrt{3}} \]

\[ S = 0.707 \]