**Problem B.5-11**

Determine a value for $K$ such that the damping ratio, $\xi = 0.5$. The find $T_r$, $T_p$, $\%OS$ (MP), and $T_s$ for a unit step input.

$G_1(s) = \frac{\frac{16}{s + 0.8}}{1 + \frac{K}{s + 0.8}} = \frac{16}{s + 0.8} + K(s + 0.8)$

$G_2(s) = \frac{\frac{16}{s(0.8 + 16K)}}{s + (0.8 + 16K)}$

$T(s) = \frac{\frac{16}{s(0.8 + 16K)}}{1 + \frac{16}{s(0.8 + 16K)}} = \frac{16}{s^2 + (0.8 + 16K)s + 16}$

$\omega_n^2 = 16$

$\omega_n = 4 \iff \frac{25\omega_n^2 - 0.8}{16} = K$

$2(0.5\times 4) - 0.8 = K$

$K = 0.2$
**Problem 8-5-11 Cont.**

\[ T(s) = \frac{16}{s^2 + (0.8 + 10)(0.2)s + 16} = \frac{16}{s^2 + 4s + 16} \]

\[ W_H = 4 \]

\[ S = 0.5 \]

\[ T_0S = M_p = e^{-\frac{s}{\sqrt{1.84}}} \times 100 \]

\[ T_0S = M_p = 16.37\% \]

\[ T_1 = \frac{\pi}{\omega_n \sqrt{1 - S^2}} = \frac{\pi}{4.96} \]

\[ T_2 = 0.915sec \]

\[ T_3 = \frac{4}{S\omega_n} = \frac{4}{0.5} = \frac{4}{0.5} \]

\[ T_3 = 7sec \]

**Tr Various Methods**

\[ T_{12} = \frac{\pi - P}{\omega_n} \Rightarrow P = \tan^{-1} \left( \frac{4}{0.5} \right) = 89.47^\circ \]

\[ T_{12} = \frac{\pi - 1.047}{3.46} \]

\[ T_{12} = 0.605 \text{ sec} \]

02. \[ T_R = \frac{4}{S\omega_n} = \frac{4}{0.5} \]

\[ T_R = 0.45 \text{ sec} \]
B-5-15

Compare the 3 systems with unit steps & unit ramps, which system is best with respect to the speed of response & maximum overshoot in the step response.

Note: Can find CLF in MATLAB as well.

**System I**

\[ T(s) = \frac{5}{\frac{s(s^2+5)}{s(s^2+1)}} = \frac{s}{s^2+5} \]

**System II**

\[ T(s) = \frac{5\left(1 + 0.8s\right)}{\frac{s(s^2+5)}{s(s^2+1)}} = \frac{s}{s^2+5} \frac{s}{s^2+1} \]

\[ T(s) = \frac{5(1+0.8s)}{5s^2+5s+5} \]

\[ T(s) = \frac{s(1+0.8s)}{s(s^2+5+4s)} \]

\[ T(s) = \frac{1+0.8s}{s^2+5+4s} \]
RESOLVING INNER F.B. LOOP

\[
\frac{5}{5s+1} = \frac{5(s+0.8)}{5s+1} = \frac{5}{5s+1+4} = \frac{5}{5s+5} = \frac{1}{s+1}
\]

\[
T(s) = \frac{\frac{1}{s+1}}{1 + \frac{1}{5(s+1)}} = \frac{1}{s^2 + 5s + 1}
\]
Controls 3708 HW #2 Solution

B-5-15 cont. (See attached plots & code)

For System I Step Response

$T_p = 1.73 \text{ sec}$

$\%OS = 72.9\%$

$T_s = 38.4 \text{ sec}$

$\xi_s = 0$

For System II Step Response

$T_p = 1.24 \text{ sec}$

$\%OS = 24.4\%$

$T_s = 7.56 \text{ sec}$

$\xi_s = 0$

For System III Step Response

$T_p = 1.165$

$\%OS = 16.3\%$

$T_s = 8.08 \text{ sec}$

$\xi_s = 0$

For System I Ramp Response: $\xi_s = 0.2$

(Takes 24.0 sec to settle)

$\text{Max Error} = 1$

(Better to use $E^* R - U$ To see)

For System II

$\xi_s = 0.2$

(Takes 8.5 sec to settle)

$\text{Max Error} = 0.048$

For System III

$\xi_s = 1$

(Takes about 8 sec to settle)

$\text{Max Error} = 1.298$
System Response for System I

Step Response for System II

Step Response for System III
Unit Ramp Response for System 1

Unit Ramp Response for System II

Unit Ramp Response for System III
%System 1
clear
sysg=tf([5],[5 1 0])
syst=feedback(sysg,[1],-1)
figure(1)
step(syst)
t=[0:0.1:60];
u=t;
[y,t]=lsim(syst,u,t);
figure(2)
plot(t,u)
hold on
plot(t,y,'--g')
title('Unit Ramp Response for System 1')
ylabel('Amplitude')
xlabel('Time (sec)')
e=u-y';
figure(3)
plot(t,e)
title('Error for Unit Ramp for System 1')
ylabel('Error')
xlabel('Time (sec)')
max(e)

%System 2
clear
sysg=tf(5*[0.8 1],[5 1 0])
syst=feedback(sysg,[1],-1)
figure(1)
step(syst)
t=[0:0.1:12];
u=t;
[y,t]=lsim(syst,u,t);
figure(2)
plot(t,u)
hold on
plot(t,y,'--g')
title('Unit Ramp Response for System 2')
ylabel('Amplitude')
xlabel('Time (sec)')
e=u-y';
figure(3)
plot(t,e)
title('Error for Unit Ramp for System 2')
ylabel('Error')
xlabel('Time (sec)')
max(e)

%System 3
clear
sysg1=tf([5],[5 1])
syst1=feedback(sysg1,[0.8],-1)
sysg2=sysg1*tf([1],[1 0])
syst2=feedback(sysg2,[1],-1)
figure(1)
step(syst2)
t=[0:0.1:12];
u=t;
[y,t]=lsim(syst2,u,t);
figure(2)
plot(t,u)
hold on
plot(t,y,'--g')
title('Unit Ramp Response for System 2')
ylabel('Amplitude')
xlabel('Time (sec)')
e=u-y';
figure(3)
plot(t,e)
title('Error for Unit Ramp for System 2')
ylabel('Error')
xlabel('Time (sec)')
max(e)
Problem 3 - 5-17

Using MATLAB, obtain a unit step response curve for the unity-feedback control system whose open loop T.F is \( G(s) = \frac{10}{s(s^2+5s+14)} \)

Using MATLAB, obtain the rise time, peak time, maximum overshoot & settling time in the unit-step response curve

\[ T(s) = \frac{G}{1 + GM} = \frac{10}{s(s^2+5s+14)} = \frac{10}{s^3+6s^2+8s+10} \]

or use

\[ \text{sys} = tf([10],
\[ \text{conv([1 2 0 3],[1 4])}
\[ \text{syst = feedback(sys, [1,1])}
\[ \text{step(sys)} \]

\[ T_r = 6 \text{ sec} \]
\[ T_p = 2.57 \text{ sec} \]
\[ \%OS = 21.47\% \]
\[ T_s = 1.12 \text{ sec} \]
Problem B-5-19

Given \( \frac{C(s)}{R(s)} = \frac{5s + 1}{s^2 + 25s + 1} \)

Where \( s = 0.2, 0.4, 0.6, 0.8, 1.0 \)

Plot a 3-D diagram of the unit-step response curves.

See attached plot & code.
clear
z=[0.2, 0.4, 0.6, 0.8, 1];
t=[0:0.2:10];
for n=1:max(size(z))
    t=[0:0.2:10];
    syst=tf([1 1],[1 2*z(n) 1])
    [y(1:51,n),x,t]=step(syst,t);
end
t=[0:0.2:10];
mesh(t,z,y')
title('3-D Plot of Unit-Step Response Curves')
xlabel('t Sec')
ylabel('zeta')
ylabel('Response')

% t was resetting I believe as a result of the step command
% so having multiple assignments was necessary - this is not
% usually the case.
Consider the unity-feedback control system with the following open-loop transfer function:

\[ G(s) = \frac{10}{s(s-1)(s+3)} \]

Is the system stable?

\[ T(s) = \frac{\frac{10}{s(s-1)}}{1 + \frac{10}{s(s-1)(s+3)}} = \frac{10}{s(s-1)(s+3) + 10} \]

\[ = \frac{10}{s^3 + s^2 - 2s + 3} + \frac{10}{s^3 + s^2 - 2s + 3} + 10 \]

Closed-loop poles @ \(-2, 2, 1.24\)

- 2 poles in RHP
- So \underline{unstable}
Consider the satellite attitude control system shown. The output of this system exhibits continued oscillations & is not desired. The system can be stabilized using tachometer feedback. If \( \frac{K}{J} = 4 \) what value of \( K_n \) will yield the damping ratio to be 0.6.

Without Tach F.B.:

\[
T(s) = \frac{K}{Js^2} \cdot \frac{s}{1 + \frac{K}{Js^2}} = \frac{K}{Js^2 + K} (\text{Not Nec for Student, Soln 1})
\]

\[
\frac{K}{Js^2 + K} = \frac{K_0}{S^2 + K_0/S}
\]

poles @ \( \pm j\sqrt{K_0} \)

so marginally stable

With Tach F.B.:

\[
\begin{align*}
R(s) & \rightarrow K \frac{1}{Js} \frac{1}{S} \rightarrow C(s) \\
& \rightarrow K_n \frac{1}{Js+K_n} \frac{1}{S} \rightarrow C(s)
\end{align*}
\]

Resolving inner F.B. loop

\[
\frac{K}{Js^2} \frac{1}{1 + \frac{K}{Js} \frac{1}{S}} = \frac{K}{Js + K_n}
\]
\[ T(s) = \frac{K}{s(Js + KK_n)} \left( 1 + \frac{K}{s(Js + KK_n)} \right) = \frac{K}{Js^2 + KK_n s + K} \]

\[ T(s) = \frac{K/J}{s^2 + (K/J)K_n s + K/J} \]

w/ \( K/J = 4 \)

\[ T(s) = \frac{4}{s^2 + 4K_n s + 4} \]

\[ \omega_n = 2 \Rightarrow \omega_n = 2 \]

\[ 4K_n = 25 \omega_n \quad \text{for} \ s = 0.6 \]

\[ K_n = \frac{25 \omega_n}{4} = \frac{2 \cdot 0.6 \times 4}{4} \]

\[ K_n = 0.6 \]
B-5-32. Consider the system shown below.

From the diagram we obtain

\[ \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) G_c(s)} \]

For a ramp disturbance \( d(t) = at \), we have \( D(s) = a/s^2 \). Hence,

\[ C(s) = \lim_{s \to 0} sC(s) = \lim_{s \to 0} \frac{s^2 G(s)}{1 + G(s) G_c(s)} \frac{a}{s^2} = \lim_{s \to 0} \frac{a}{s G_c(s)} \]

\( c(\infty) \) becomes zero if \( G_c(s) \) contains double integrators.
HW #2 Solution

B-5-31 (Not Assigned, but intended for too)

Consider a unity feedback control system whose open loop transfer function is \( G(s) = \frac{k}{s(js + b)} \)

Discuss the effects that varying the values of \( k \) & \( b \) has on the steady state error in a unit ramp response. Sketch typical unit ramp response curves for a small value, medium value, and large value of \( k \) assuming \( b \) is constant.

Unity F.B

Type 1 System

So for a unit ramp \( e_{ss} = \frac{1}{k} \)

\[ K_u = \lim_{s \to 0} sG(s) = \lim_{s \to 0} s \left( \frac{k}{s(js + b)} \right) = \frac{k}{b} \]

\[ e_{ss} = \frac{L}{b} = \frac{B}{K} = e_{ss} \text{ constant} \]

As \( K \) increases, \( e_{ss} \) is less
As \( b \) increases, \( e_{ss} \) is more

See plot \( u/b = 1, j = 1 \& k = 1, 2, 4 \)