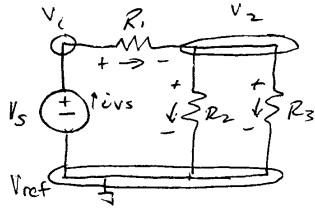


# EE301/302 S.P.09 HW#3 SOLUTION 4

## PROBLEM 1



GIVEN:  $V_s = 30V$ ,  $R_1 = R_2 = R_3 = 500\Omega$

- FIND:
- ALL NODE VOLTAGES
  - $V_{R1}, V_{R2}, V_{R3}$
  - $i_{R1}, i_{R2}, i_{R3}$
  - $I_{V_s}$

- ? KNOWN & UNKNOWN NODE VOLTAGES

$$V_1 = V_s \quad V_1 \text{ KNOWN}$$

$$V_2 \text{ UNK.} \quad \underline{V_1 = 30V \leftarrow}$$

KCL NR2

$$i_1 - i_2 - i_3 = 0$$

$$\frac{V_1 - V_2}{R_1} - \left[ \frac{V_2}{R_2} \right] - \left[ \frac{V_2}{R_3} \right] = 0$$

$$\frac{V_s}{R_1} = \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] V_2$$

$$V_2 = \frac{V_s}{R_1} \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right]^{-1}$$

$$V_2 = \frac{30}{500} \left[ \frac{1}{500} + \frac{1}{500} + \frac{1}{500} \right]^{-1}$$

$$= \frac{30}{500} \left[ \frac{3}{500} \right]^{-1} = \frac{30}{500} \left[ \frac{500}{3} \right]$$

$$\underline{V_2 = 10V \leftarrow}$$

$$V_{R1} = V_1 - V_2 = 30 - 10$$

$$\underline{V_{R1} = 20V \leftarrow}$$

$$V_{R2} = V_2 - V_{ref} = 10 - 0$$

$$\underline{V_{R2} = 10V \leftarrow}$$

$$V_{R3} = V_2 - V_{ref} = 10 - 0$$

$$\underline{V_{R3} = 10V \leftarrow}$$

$$i_{R1} = \frac{V_1 - V_2}{R_1} = \frac{30 - 10}{500}$$

$$\underline{i_{R1} = 0.04A = 40mA \leftarrow}$$

$$i_{R2} = \frac{V_2}{R_2} = \frac{10}{500}$$

$$\underline{i_{R2} = 0.02A = 20mA \leftarrow}$$

$$i_{R3} = \frac{V_2}{R_3} = \frac{10}{500}$$

$$\underline{i_{R3} = 0.02A = 20mA \leftarrow}$$

N1 KCL

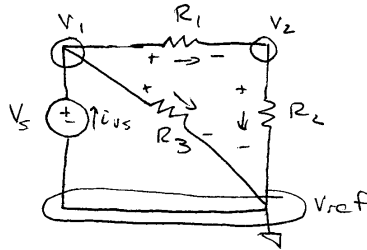
$$i_{v_s} - i_1 = 0$$

$$i_{v_s} = i_1$$

$$\underline{i_{v_s} = 0.04A = 40mA \leftarrow}$$

EE301/303 SP09 HW#3 SOLUTION

PROBLEM 2



Given:  $V_s = 30V$ ,  $R_1 = 100\Omega$   
 $R_2 = 200\Omega$ ,  $R_3 = 300\Omega$

Find: • All Node Voltages

•  $V_{R1}, V_{R2}, V_{R3}$

•  $i_{R1}, i_{R2}, i_{R3}$

•  $i_{Vs}$

$V_1 = V_s$

$V_1 = 30V \leftarrow$

$V_2$  IS UNK

KCL N2

$i_1 - i_2 = 0$

$\left[ \frac{V_1 - V_2}{R_1} \right] - \left[ \frac{V_2 - V_{ref}}{R_2} \right] = 0$

$\frac{V_s}{R_1} = \left[ \frac{1}{R_1} + \frac{1}{R_2} \right] V_2$

$\frac{V_s}{R_1} = \frac{R_2 + R_1}{R_1 R_2} V_2$

$V_2 = \frac{R_2}{R_1 + R_2} \cdot V_s$

$V_2 = \frac{200}{100 + 200} \cdot 30$

$= \frac{200 \cdot 30}{300}$

$V_2 = 20V \leftarrow$

$V_{R1} = V_1 - V_2 = 30 - 20$

$V_{R1} = 10V \leftarrow$

$V_{R2} = V_2 - V_{ref} = V_2$

$V_{R2} = 20V \leftarrow$

$V_{R3} = V_1 - V_{ref} = V_1$

$V_{R3} = 30V \leftarrow$

$i_{R1} = \frac{V_1 - V_2}{R_1} = \frac{30 - 20}{100} = \frac{10}{100}$

$i_{R1} = 0.1A = 100mA \leftarrow$

$i_{R2} = \frac{V_2 - V_{ref}}{R_2} = \frac{V_2}{R_2} = \frac{20}{200}$

$i_{R2} = 0.1A = 100mA \leftarrow$

$i_{R3} = \frac{V_1 - V_{ref}}{R_3} = \frac{V_1}{R_3} = \frac{30}{300}$

$i_{R3} = 0.1A = 100mA \leftarrow$

KCL N1

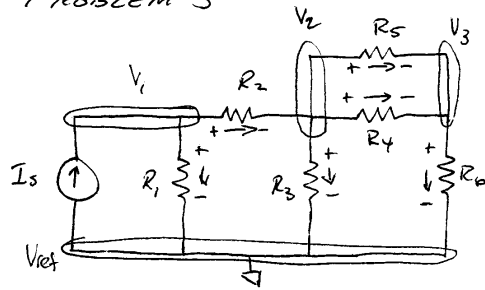
$i_{Vs} - i_3 - i_1 = 0$

$i_{Vs} = i_1 + i_3 = 0.1 + 0.1$

$i_{Vs} = 0.2A = 200mA \leftarrow$

EE301/303 SPO9 HW #3 SOLUTION

PROBLEM 3



$V_1, V_2, V_3$  UNK.

Given:  $R_1 = 20\Omega, R_2 = 10\Omega$   
 $R_3 = R_4 = R_5 = 20\Omega$   
 $R_6 = 30\Omega, I_S = 5A$

FIND: • ALL NODE VOLTAGES  
 •  $i_{R3}, i_{R4}$   
 •  $V_{R5}$   
 •  $V_{I_S}$

KCL N1

$$I_S - i_1 - i_2 = 0$$

$$I_S - \left[ \frac{V_1 - V_{ref}}{R_1} \right] - \left[ \frac{V_1 - V_2}{R_2} \right] = 0$$

$$I_S = \left[ \frac{1}{R_1} + \frac{1}{R_2} \right] V_1 - \left[ \frac{1}{R_2} \right] V_2 \quad \leftarrow \text{Egn. 1}$$

$$i_{R3} = \frac{V_2 - V_{ref}}{R_3}$$

$$i_{R3} = 1.54A \quad \leftarrow$$

$$i_{R4} = \frac{V_2 - V_3}{R_4}$$

$$i_{R4} = 0.3846A \quad \leftarrow$$

KCL N2

$$i_2 - i_3 - i_4 - i_5 = 0$$

$$\left[ \frac{V_1 - V_2}{R_2} \right] - \left[ \frac{V_2 - V_{ref}}{R_3} \right] - \left[ \frac{V_2 - V_3}{R_4} \right] - \left[ \frac{V_2 - V_3}{R_5} \right] = 0$$

$$\left[ \frac{1}{R_2} \right] V_1 - \left[ \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right] V_2 + \left[ \frac{1}{R_4} + \frac{1}{R_5} \right] V_3 = 0 \quad \leftarrow \text{Egn. 2}$$

$$V_{R5} = V_2 - V_3$$

$$V_{R5} = 7.69V \quad \leftarrow$$

$$V_{I_S} = V_1$$

$$V_{I_S} = 53.85V \quad \leftarrow$$

KCL N3

$$i_4 + i_5 - i_6 = 0$$

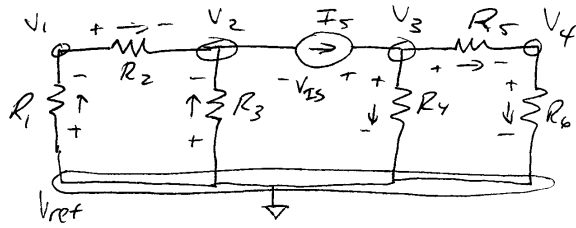
$$\left[ \frac{V_2 - V_3}{R_4} \right] + \left[ \frac{V_2 - V_3}{R_5} \right] - \left[ \frac{V_3 - V_{ref}}{R_6} \right] = 0$$

$$\left[ \frac{1}{R_4} + \frac{1}{R_5} \right] V_2 - \left[ \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6} \right] V_3 = 0 \quad \leftarrow \text{Egn. 3}$$

SOLVING EGNS:  $V_1 = 53.85V$   
 $V_2 = 30.77V$   
 $V_3 = 23.07V \quad \leftarrow$

# EE301/303 SP09 HW#3 SOLUTIONS

## PROBLEM 4 (REVISED)



Given:  $R_1 = 15\Omega$ ,  $R_2 = 15\Omega$   
 $R_3 = 30\Omega$ ,  $R_4 = 40\Omega$   
 $R_5 = 20\Omega$ ,  $R_6 = 20\Omega$   
 $I_5 = 100\text{ mA}$

FIND:  
 • All Node Voltages  
 •  $i_{R2}$ ,  $i_{R4}$ ,  $V_{R5}$ ,  $V_{R3}$   
 •  $V_{I5}$

### KCL N1

$$i_1 - i_2 = 0$$

$$\left[ \frac{V_{ref} - V_1}{R_1} \right] - \left[ \frac{V_1 - V_2}{R_2} \right] = 0$$

$$\boxed{-\left[ \frac{1}{R_1} + \frac{1}{R_2} \right] V_1 + \left[ \frac{1}{R_2} \right] V_2 = 0} \quad \text{Egn 1}$$

### KCL N2

$$i_2 + i_3 - I_5 = 0$$

$$\left[ \frac{V_1 - V_2}{R_2} \right] + \left[ \frac{V_{ref} - V_2}{R_3} \right] - I_5 = 0$$

$$\boxed{\left[ \frac{1}{R_2} \right] V_1 - \left[ \frac{1}{R_2} + \frac{1}{R_3} \right] V_2 = I_5} \quad \text{Egn 2}$$

### KCL N3

$$I_5 - i_4 - i_5 = 0$$

$$I_5 - \left[ \frac{V_3 - V_{ref}}{R_4} \right] - \left[ \frac{V_3 - V_4}{R_5} \right] = 0$$

$$\boxed{I_5 = \left[ \frac{1}{R_4} + \frac{1}{R_5} \right] V_3 - \left[ \frac{1}{R_5} \right] V_4} \quad \text{Egn 3}$$

### KCL N4

$$i_5 - i_6 = 0$$

$$\left[ \frac{V_3 - V_4}{R_5} \right] - \left[ \frac{V_4 - V_{ref}}{R_6} \right] = 0$$

$$\boxed{\left[ \frac{1}{R_5} \right] V_3 - \left[ \frac{1}{R_5} + \frac{1}{R_6} \right] V_4 = 0} \quad \text{Egn 4}$$

Solving Egn's:

$$V_1 = -0.75\text{ V}$$

$$V_2 = -1.5\text{ V}$$

$$V_3 = 2.0\text{ V}$$

$$V_4 = 1.0\text{ V}$$

$$i_{R2} = \frac{V_1 - V_2}{R_2}$$

$$i_{R2} = 0.05\text{ A} = 50\text{ mA} \rightarrow$$

$$i_{R4} = \frac{V_3 - V_{ref}}{R_4}$$

$$i_{R4} = 0.05\text{ A} = 50\text{ mA} \downarrow$$

$$V_{R5} = V_3 - V_4$$

$$V_{R5} = 1\text{ V} \leftarrow$$

$$V_{R3} = V_{ref} - V_2 = -V_2$$

$$V_{R3} = 1.5\text{ V} \leftarrow$$

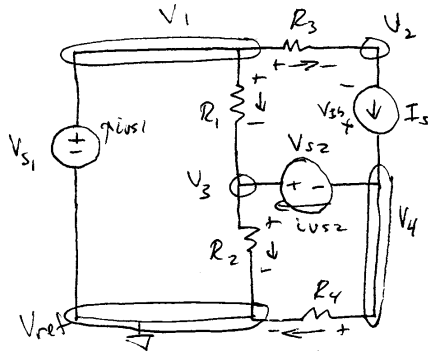
$$V_{I5} = V_3 - V_2$$

$$= 2 - (-1.5)$$

$$V_{I5} = 3.5\text{ V} \leftarrow$$

EE301/303 SPO9 HW#3 SOLUTION

PROBLEM 5



Given:  $R_1 = R_2 = 10k\Omega$   
 $R_3 = 2k\Omega, R_4 = 1k\Omega$   
 $V_{S1} = 12V, V_{S2} = 0.5V$   
 $I_S = 2.5mA$

FIND: • ALL NODE VOLTAGES  
 •  $i_{R1}, i_{R2}, i_{R3}, i_{R4}$   
 •  $i_{V_{S1}}, i_{V_{S2}}$   
 •  $I_{IS}$

KNOWN:  $V_1 \rightarrow V_1 = V_{S1} = 12V = V_1 \leftarrow$

UNK:  $V_2, V_3, V_4$

$V_3, V_4$  DEPENDENT  
 $V_3 - V_4 = V_{S2} \leftarrow \text{EQU 1}$

SOLVING EQNS:  $V_3 = 3.5V \leftarrow$   
 $V_4 = 3V \leftarrow$

$i_{R1} = \frac{V_1 - V_3}{R_1} \leftarrow$

$i_{R1} = 8.5 \times 10^{-4} A = 0.85mA \downarrow \leftarrow$

$i_{R4} = \frac{V_2 - V_4}{R_4} \rightarrow i_{R4} = 0.003A = 3mA \leftarrow$

$V_{R3} = V_1 - V_2$   
 $V_{R3} = 5V \leftarrow$

$V_{R2} = V_3$   
 $V_{R2} = 3.5V \leftarrow$

$V_{IS} = V_2 - V_3 \rightarrow V_{IS} = -4V \leftarrow$

KCL N1:  $i_{V_{S1}} - i_{R1} - i_{R3} = 0$

$i_{V_{S1}} = \left[ \frac{V_1 - V_3}{R_1} \right] - \left[ \frac{V_1 - V_2}{R_3} \right]$

$i_{V_{S1}} = 0.0034A \approx 3.4mA \leftarrow$

KCL N4  $\rightarrow i_{V_{S2}} = I_S - i_{R4}$

$= I_S - \frac{V_2 - V_4}{R_4}$

$i_{V_{S2}} = -5 \times 10^{-4} A$   
 $= 0.5mA \leftarrow$

KCL N2

$i_3 - I_S = 0$   
 $\left[ \frac{V_1 - V_2}{R_3} \right] - I_S = 0$

$-\left[ \frac{1}{R_3} \right] V_2 = I_S - \frac{V_{S1}}{R_3}$

$V_2 = -R_3 \left[ I_S - \frac{V_{S1}}{R_3} \right]$

$V_2 = -R_3 I_S + V_{S1}$

$V_2 = -(2000)(0.0025) + 12$

$V_2 = 7V \leftarrow$

KCL N3

$i_1 - i_2 + i_{V_{S2}} = 0$

KCL N4

$I_S - i_{V_{S2}} - i_4 = 0$

$i_{V_{S2}} = I_S - i_4$

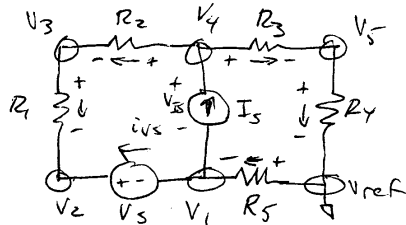
COMBINED N3/N4 (OR SUPERNODE)

$i_1 - i_2 + I_S - i_4 = 0$

$\left[ \frac{V_1 - V_3}{R_1} \right] - \left[ \frac{V_3 - V_{ref}}{R_2} \right] + I_S - \left[ \frac{V_2 - V_{ref}}{R_4} \right] = 0$

$\frac{V_{S1}}{R_1} + I_S = \left[ \frac{1}{R_1} + \frac{1}{R_2} \right] V_3 + \left[ \frac{1}{R_4} \right] V_4 \leftarrow \text{EQU 2}$

PROBLEM 6



GIVEN:  $V_s = 12V$ ,  $I_s = 5mA$   
 $R_1 = 300\Omega$ ,  $R_2 = 500\Omega$ ,  $R_3 = 60\Omega$   
 $R_4 = 240\Omega$ ,  $R_5 = 180\Omega$

FIND: ALL THE NODE VOLTAGES

- $i_{R2}$ ,  $i_{R3}$ ,  $V_{R1}$ ,  $V_{R4}$
- $i_{Vs}$ ,  $V_{is}$

$V_1, V_2, V_3, V_4, V_5$  GUNK

$V_1, V_2$  DEP  
 $V_2 - V_1 = V_s$  EQU 1

KCL N1  
 $i_s - i_{Vs} - I_s = 0$   
 KCL N2  
 $i_{Vs} + i_1 = 0$   
 $i_{Vs} = -i_1$

KCL N1/N2 COMBINED (SN EQU)

$i_s + i_1 - I_s = 0$

$\left[\frac{V_{ref} - V_1}{R_5}\right] + \left[\frac{V_3 - V_2}{R_1}\right] - I_s = 0$

$I_s = -\left[\frac{1}{R_5}\right]V_1 - \left[\frac{1}{R_1}\right]V_2 + \left[\frac{1}{R_1}\right]V_3$  EQU 2

KCL N3

$-i_1 + i_2 = 0$

$-\left[\frac{V_3 - V_2}{R_1}\right] + \left[\frac{V_4 - V_3}{R_2}\right] = 0$

$\left[\frac{1}{R_1}\right]V_2 - \left[\frac{1}{R_1} + \frac{1}{R_2}\right]V_3 + \left[\frac{1}{R_2}\right]V_4 = 0$  EQU 3

KCL N4

$I_s - i_2 - i_3 = 0$

$I_s - \left[\frac{V_4 - V_3}{R_2}\right] - \left[\frac{V_4 - V_5}{R_3}\right] = 0$

$I_s = \left[\frac{1}{R_2}\right]V_3 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right]V_4 - \left[\frac{1}{R_3}\right]V_5$  EQU 4

KCL N5

$i_3 - i_4 = 0$

$\left[\frac{V_4 - V_5}{R_3}\right] - \left[\frac{V_5 - V_{ref}}{R_4}\right] = 0 \rightarrow \left[\frac{1}{R_3}\right]V_4 - \left[\frac{1}{R_3} + \frac{1}{R_4}\right]V_5 = 0$  EQU 5

Solving EQU

$V_1 = -2.25V$   
 $V_2 = 9.75V$   
 $V_3 = 7.5V$   
 $V_4 = 3.75V$   
 $V_5 = 3V$

$i_{R2} = \frac{V_4 - V_3}{R_2}$

$i_{R2} = -0.0075A$

$i_{R3} = \frac{V_4 - V_5}{R_3}$

$i_{R3} = 0.0125A$

$V_{R1} = V_3 - V_2$

$V_{R1} = -2.25V$

$V_{R4} = V_5$

$V_{R4} = 3V$

$i_{Vs} = -i_{R1} = -\frac{(V_3 - V_2)}{R_1}$

$i_{Vs} = 0.0075A$

$V_{Is} = V_4 - V_1$

$V_{Is} = 6V$