Name Solution

Digital Controls EE651 Feb. 6, 2009
Quiz 1

Find the inverse z-transform of $X(z) = \frac{0.2}{(z-1)(z-0.8)}$ using partial fraction expansion. Find x(k) for k = 0, 1, 2, 3 and verify the answer using direct division.

$$\frac{X(z)}{z} = \frac{0.2}{z(z-1)(z-0.8)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-0.8}$$

$$A = \frac{0.2}{(z-1)(z-0.8)}\Big|_{z=0} = \frac{0.2}{(-1)(-0.8)} = \frac{1}{4}$$

$$B = \frac{0.2}{z(z-0.8)}\Big|_{z=1} = \frac{0.2}{(1)(0.2)} = 1$$

$$C = \frac{0.2}{z(z-1)}\Big|_{z=0.8} = \frac{0.2}{(0.8)(0.8-1)} = -\frac{5}{4}$$

$$\frac{X(z)}{z} = \frac{0.25}{z} + \frac{1}{z-1} - \frac{1.25}{z-0.8}$$

$$X(z) = 0.25 + \frac{z}{z-1} - \frac{z(1.25)}{z-0.8}$$

$$\mathcal{Z}^{-1}[0.25] = \begin{cases} 0.25 & k=0 \\ 0 & k \neq 0 \end{cases}$$

$$\mathcal{Z}^{-1}\left[\frac{z}{z-1}\right] = \begin{cases} 1 & k \geq 0 \\ 0 & k < 0 \end{cases}$$

$$\mathcal{Z}\left[\frac{z}{z-0.8}\right] = \begin{cases} 0.8^k & k > 0 \\ 0 & k < 0 \end{cases}$$

$$x(k) = \begin{cases} 0.25 + 1 - 1.25(0.8)^0 & k=0 \\ 1 - 1.25(0.8)^k & k > 0 \end{cases} \rightarrow \boxed{\begin{cases} 0 & k=0 \\ 1 - 1.25(0.8)^k & k > 0 \end{cases}}$$

$k=0$; $x(0) = 0$
$k=1$ $x(1) = 1 - 1.25(0.8)^1 = 0$
$k=2$ $x(2) = 1 - 1.25(0.8)^2 = 0.2$
$k=3$ $x(3) = 1 - 1.25(0.8)^3 = 0.36$

$$\begin{array}{r} 0.2z^{-2} + 0.36z^{-3} \\ z^2 - 1.8z + 0.8 \,\big)\, 0.2 \\ -(0.2 - 0.36z^{-1} + 0.16z^{-2}) \\ \hline +0.36z^{-1} - 0.16z^{-2} \\ -(0.36z^{-1} + 0.648z^{-2} + 0.288z^{-3}) \\ \hline +0.488z^{-2} - 0.288z^{-3} \end{array}$$

From Direct Division

$x(0) = 0$
$x(1) = 0$
$x(2) = 0.2$
$x(3) = 0.36$

ANOTHER PARTIAL FRACTION APPROACH

$$X(z) = \frac{0.2}{(z-1)(z-0.8)} = \frac{A}{z-1} + \frac{B}{z-0.8} = \frac{1}{z-1} - \frac{1}{z-0.8}$$

$$A = \left.\frac{0.2}{z-0.8}\right|_{z=1} = \frac{0.2}{0.2} = 1$$

$$B = \left.\frac{0.2}{z-1}\right|_{0.8} = \frac{0.2}{-0.2} = -1$$

$$z\,X(z) = \frac{z}{z-1} - \frac{z}{z-0.8}$$

$$X(z) = z^{-1}\left[\frac{z}{z-1}\right] - z^{-1}\left[\frac{z}{z-0.8}\right]$$

$$X(z) = z^{-1}\left[\left[\frac{z}{z-1}\right] - \left[\frac{z}{z-0.8}\right]\right]$$

$$X(kT) = z^{-1}\left[1 - 0.8^{k}\right]$$

DELAY OPERATOR SO DELAYS BY 1T

$$\boxed{x(k) = \begin{cases} 0 & k \le 0 \\ 1-0.8^{(k-1)} & k \ge 1 \end{cases}}$$

$$\boxed{\begin{aligned} x(0) &= 0 \\ x(1) &= 0 \\ x(2) &= 0.2 \\ x(3) &= 0.36 \end{aligned}}$$

NOT REQUIRED

INVERSION FORMULA METHOD

$$X(z) = \frac{0.2}{(z-1)(z-0.8)}$$

$$x(kT) = K_1 + K_2$$

$$K_1 = \text{Residue @ } z=1 \text{ of } \left.\left[\frac{(z-1)(0.2)z^{k-1}}{(z-1)(z-0.8)}\right]\right|_{z=1}$$

$$K_1 = \frac{0.2(1)^{k-1}}{0.2} = 1 = K_1$$

$$K_2 = \text{Residue @ } z=0.8 \text{ of } \left.\left[\frac{(z-\cancel{0.8})(0.2)z^{k-1}}{(z-1)(z-\cancel{0.8})}\right]\right|_{z=0.8}$$

$$K_2 = \frac{0.2(0.8)^{(k-1)}}{(0.8-1)} = -0.8^{(k-1)}$$

$$\boxed{X(kT) = 1 - 0.8^{(k-1)} \qquad k \ge 0}$$

FOR K=0 CASE

$$z^{(k-1)}X(z) = \frac{(0.2)}{z(z-1)(z-0.8)} \rightarrow X(kT) = k_1 + k_2 + k_3 \Rightarrow x(0) = \text{Residues of } z^{k}X(z) \text{ for poles}$$

Residue @ $z=0$

$$\left.\frac{\cancel{z}\,.\,0.2}{\cancel{z}(z-1)(z-0.8)}\right|_{z=0} = \frac{0.2}{(-1)(-0.8)} = \frac{1}{4}$$

Residue @ $z=1$

$$\left.\frac{\cancel{(z-1)}(0.2)}{z\cancel{(z-1)}(z-0.8)}\right|_{z=1} = \frac{0.2}{(1)(0.2)} = 1$$

Residue @ $z=0.8$

$$\left.\frac{\cancel{(z-0.8)}(0.2)}{z(z-1)\cancel{(z-0.8)}}\right|_{0.8} = \frac{0.2}{(0.8)(-0.2)} = \frac{-5}{4}$$

$$x(0) = \frac{1}{4} + 1 - \frac{5}{4} = 0$$

$$\boxed{x(0) = 0}$$