Name SOLUTION

Digital Controls EE651 Feb. 27, 2009
Quiz 2

Assume that a digital filter is given by the following difference equation:

y(k) + 2y(k-1) +3y(k-2)= 0.5x(k)+0.25x(k-1)

Draw block diagrams for the filter using direct programming (delays separate) and standard programming (delays combined).

DIRECT PROGRAMMING

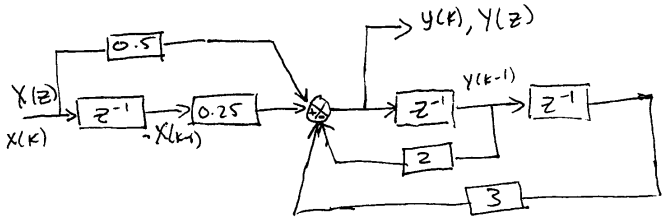
$$y(k) + 2y(k-1) + 3y(k-2) = 0.5x(k) + 0.25x(k-1)$$

$$Y(z) + 2z^{-1}Y(z) + 3z^{-2}Y(z) = 0.5X(z) + 0.25z^{-1}X(z)$$

$$Y(z)\left(1 + 2z^{-1} + 3z^{-2}\right) = X(z)\left(0.5 + 0.25z^{-1}\right)$$

$$\frac{Y(z)}{X(z)} = \frac{0.5 + 0.25z^{-1}}{1 + 2z^{-1} + 3z^{-2}}$$

$$\rightarrow y(k) = 0.5x(k) + 0.25x(k-1) - 2y(k-1) - 3y(k-2)$$



STANDARD PROGRAMMING

$$G(z) = \frac{Y(z)}{H(z)} \cdot \frac{H(z)}{X(z)} = 0.5 + 0.25z^{-1} \cdot \frac{1}{1 + 2z^{-1} + 3z^{-2}}$$

$$\frac{Y(z)}{H(z)} = 0.5 + 0.25z^{-1}$$

$$Y(z) = \left(0.5 + 0.25z^{-1}\right)H(z)$$

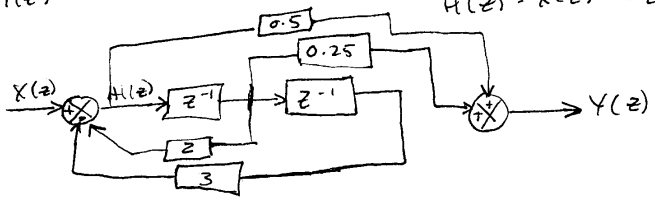
$$Y(z) = 0.5H(z) + 0.25z^{-1}H(z)$$

$$\frac{H(z)}{X(z)} = \frac{1}{1 + 2z^{-1} + 3z^{-2}}$$

$$X(z) = H(z)\left(1 + 2z^{-1} + 3z^{-2}\right)$$

$$X(z) = H(z) + 2z^{-1}H(z) + 3z^{-2}H(z)$$

$$H(z) = X(z) - 2z^{-1}H(z) - 3z^{-2}H(z)$$



VIA LADDER PROGRAMMING

$$\frac{0.5+0.25z^{-1}}{1+2z^{-1}+3z^{-2}} = \frac{0.5z^2+0.25z}{z^2+2z+3} =$$

GOAL $G(z) = A_0 + \cfrac{1}{B_1 z + \cfrac{1}{A_1 + \cfrac{1}{B_2 z + \cfrac{1}{A_2}}}}$

$$= \frac{0.5(z^2+2z+3) - 0.5(2)z - 0.5(3) + 0.25z}{z^2+2z+3} = \frac{0.5(z^2+2z+3)}{z^2+2z+3} + \frac{(-z-1.5+0.25z)}{z^2+2z+3}$$

$$= 0.5 + \frac{-\frac{3}{4}z - \frac{3}{2}}{z^2+2z+3} = 0.5 + \cfrac{1}{\cfrac{z^2+2z+3}{-\frac{3}{4}z-\frac{3}{2}}}$$

TO REDUCE $\rightarrow -\frac{3}{4}z - \frac{3}{2}$ into z^2+2z+3:

$$\begin{array}{r} -\frac{4}{3}z \\ \hline z^2+2z+3 \\ -(z^2+2z\ ) \\ \hline +3 \end{array}$$

$$= 0.5 + \cfrac{1}{-\frac{4}{3}z + \cfrac{3}{-\frac{3}{4}z-\frac{3}{2}}} = 0.5 + \cfrac{1}{-\frac{4}{3}z + \cfrac{1}{\frac{-\frac{3}{4}z-\frac{3}{2}}{3}}}$$

$$= 0.5 + \cfrac{1}{-\frac{4}{3}z + \cfrac{1}{-\frac{1}{4}z + \frac{1}{-2}}}$$

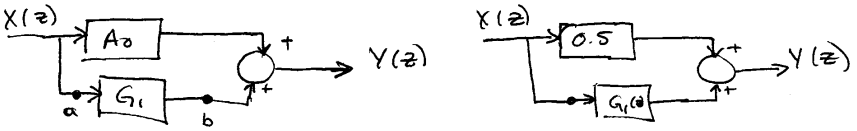
SINCE z TERM ZEROED A_1 TERM WILL NOT BE PRESENT

EE651 SP09 Quiz 2 Extended to Ladder Prog.

GOAL: $G(z) = A_0 + \cfrac{1}{B_1 z + \cfrac{1}{A_1 + \cfrac{1}{B_2 z + \cfrac{1}{A_2}}}}$

$$G(z) = A_0 + G_1(z) = \frac{Y(z)}{X(z)} \rightarrow Y(z) = A_0 X(z) + G_1(z) X(z)$$

Block Diagram



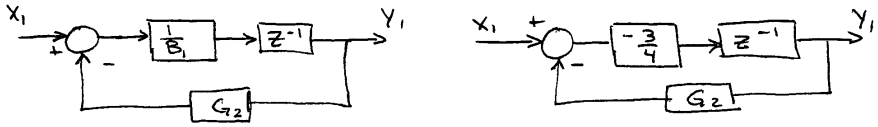
$$G_1 = \frac{Y_1(z)}{X_1(z)} = \frac{1}{B_1 z + G_2}$$

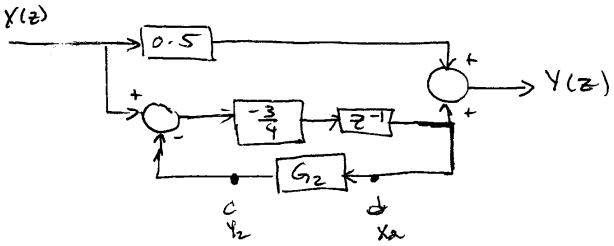
$$X_1(z) = Y_1(z) B_1 z + G_2 Y_1(z)$$

$$Y_1(z) = \frac{1}{B_1 z} X_1(z) - \frac{G_2(z)}{B_1 z} Y_1(z)$$

$$Y_1(z) = \frac{1}{B_1} z^{-1} X_1(z) - \frac{1}{B_1} z^{-1} G_2(z) Y_1(z)$$

Structure Between a & b





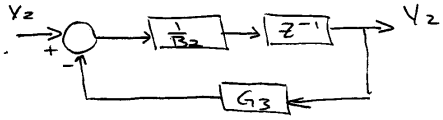
EE651 SP09 Quiz 2 Extended To Ladder Prog.

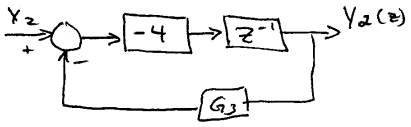
$$G_2 = \frac{1}{B_2 z + G_3} = \frac{Y_2(z)}{X_2(z)}$$

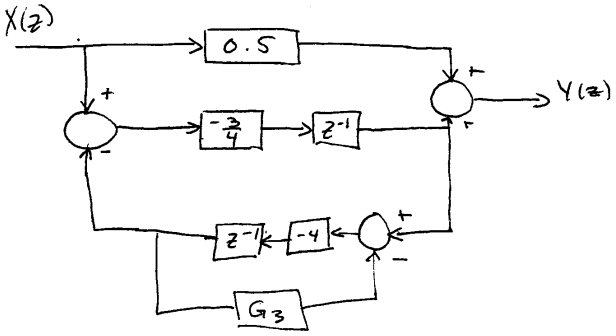
$$X_2(z) = Y_2(z)(B_2 z + G_3)$$

$$X_2(z) = Y_2(z) B_2 z + G_3 Y_2(z)$$

$$Y_2(z) = \frac{1}{B_2} z^{-1} X_2(z) - \frac{1}{B_2} z^{-1}$$







$$G_3 = \frac{1}{A_2} = \frac{1}{-2}$$