Math 315 Notes for Test 3 4/17/09

Test 3 will cover Chapter 3, the supplement on Fourier approximations, and our brief treatment of Chapter 4. You may bring a sheet of notes with your favorite formulas on it.

The format will be similar to what you have seen on the first two exams: there will likely be a set of True / False questions over the ideas of these chapters, and there will certainly be problems covering the computations. As before, it is very possible that you will see problems directly from the homework.

Here are the highlights of this material:

In $\mathbb{R}^n$: Know how to compute length and inner products. What does it mean for vectors to be orthogonal? Know the definition of orthogonal subspaces. Understand why the row space and nullspace of a matrix must be orthogonal.

Find the projection of a vector $b$ onto the line in the direction of vector $a$ (box 3H p. 154).

One of the two great ideas in Chapter 3 is the least squares solution to $Ax = b$. You should have a thorough understanding of the development of the least squares normal equations $A^TA\tilde{x} = A^Tb$ (look in your class notes or see pages 161-162). This development is very geometric in nature: if $b$ is not in the column space of $A$, find the linear combination of columns $A\tilde{x}$ that brings you closest to $b$. And of course, you should be able to set up and solve computational problems of this nature.

The Gram Schmidt process converts a basis for a subspace of $\mathbb{R}^n$ to an orthonormal basis for the same subspace. Be able to do the computations to go through all or part of the process. Understand the fundamental properties of matrices with orthonormal columns, including (square) orthogonal matrices. How does the factorization $A=QR$ affect the least squares problem?

The second great idea is the Fourier approximation of $f(x)$. The essential idea is that the functions

$$\frac{1}{\sqrt{2}}, \cos x, \sin x, \cos 2x, \sin 2x \ldots \cos nx, \sin nx$$

form an orthonormal basis for a subspace of the function space $C[-\pi, \pi]$, with inner product

$$(f, g) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)dx.$$  You should be able to show that basis functions are orthogonal unit vectors, and you should be able to find Fourier coefficients $c_0, a_1, b_1, \ldots a_n, b_n$ for a given function $f$. (I’ll try to make any integrals fairly tame, but you have to understand that integration absolutely fundamental to the idea of orthogonality in this setting.)

Determinants: know how to compute small determinants by cofactors along any row or column. Understand why the cofactor method is not reasonable for large matrices. Be familiar with the determinant rules developed in class (and in section 4.2). In particular, understand how rules 5 and 7 provide a reasonable method of computing determinants of large matrices.