Math 315 Notes for Final Exam Spring 2009

Review Session Wednesday 5/6/09 7:00 pm McLaury 306

Your final exam will be given on Friday, May 8 from 9:00 am to 10:50 am, in McLaury 306 (our usual classroom). You should expect to see problems similar to homework and previous exam problems. There will certainly be questions on the material on eigenvalues covered after Exam 3. If a text section is not listed below, there will be no exam questions directly from that section. Sections marked with a * are the most critical sections of the course, and are likely to be emphasized. You may bring one sheet of notes into the exam.

1.3 Solve systems via Gaussian Elimination. What are pivots, and how are they related to singular vs. nonsingular? No questions from the “Cost of Elimination.”

1.4 Know the rules for matrix multiplication. Box 1A is a critical idea.

1.5* Factor $A$ into $LU$. Solve $Ax = b$ via $Lc = b$ and $Ux = c$. Understand the role of $P$ in $PA = LU$.

1.6 Invert $A$ as on page 47. Know rules for inverses and transposes (boxes 1L, 1M). This section starts a very long list of equivalent conditions for nonsingularity, which is continued in later sections. See the very last page of your text, “Linear Algebra in a Nutshell.”

2.1* What’s a vector space? Be able to tell if a given set of vectors is a subspace of some vector space (as in problem 2.1.2). Understand column space and nullspace of an $m \times n$ matrix; also the relationship between the column space of $A$ and existence of solutions to $Ax = b$.

2.2* Understand echelon form, basic (pivot) variables, free variables, rank, and the general solution to $Ax = 0$ and $Ax = b$, where $A$ is $m \times n$.

2.3* This section is absolutely critical! Linear independence, span, basis, and dimension are the key ideas.

2.4 Item 2Q is the only thing in this section that might show up on your exam. (We did not discuss left and right inverses.)

2.6 A linear transformation $L : V \rightarrow W$ is a function that transforms vectors into new vectors, always obeying two rules: $L(x + y) = L(x) + L(y)$ and $L(cx) = cL(x)$. A crucial fact that comes from these two rules is that a linear transformation is completely determined by what it does to a set of basis vectors for $V$. Also, every linear transformation can be represented by a matrix multiplication. The matrix of a linear transformation depends the bases chosen for $V$ and $W$. Unless otherwise stated, we use the standard basis vectors $(1, 0, \ldots, 0), (0, 1, 0, \ldots, 0)$ etc. for both $V$ and $W$.

3.1 Fundamental ideas: inner products, orthogonality, length. Nothing on orthogonal complements, orthogonal subspaces, etc.

3.3* Understand the least squares equation $A^T Ax = A^T b$ and the ideas leading up to it. Understand the geometry of the column space of $A$ and the vectors $b$ and $p$. Be able to formulate and solve least squares problems like 3.3.13, 3.3.18, and 3.3.25.

3.4 Basic ideas: orthogonal matrix, Gram-Schmidt orthogonalization, and the $QR$ factorization.

4.2 Be familiar with the basic properties of the determinant. What is the only computationally reasonable way to calculate a determinant of a large matrix?

5.1* Read this section thoroughly! Know the basic ideas in computing eigenvalues and eigenvectors; trace and determinant properties (box 5B); and eigenvalues of diagonal, upper, and lower triangular matrices.

5.2 Basic idea: factor $A$ into $A = SAS^{-1}$. Understand conditions on eigenvalues and eigenvectors that determine whether such a factorization exists. How can this factorization be used to “uncouple” a system of differential equations? Solve systems of differential equations by uncoupling them (problem 5.1.2 via the diagonalization process emphasized in class).