Agenda

Web Resources
Schedule
Factory Physics
Chapter 2: Inventory Control From EOQ to ROP conclusion
Chapter 3: The MRP Crusade
New assigned problems C3: 2,3,5,6,11

(Q,r) Example

Stocking Repair Parts:
D = 14 units per year
c = $150 per unit
h = 0.1 x 150 + 10 = $25 per unit
l = 45 days
\[ \theta = \frac{14 \times 45}{365} = 1.726 \text{ units during replenishment lead time} \]
A = $10
b = $40
Demand during lead time is Poisson

Values for Poisson(\theta) Distribution

<table>
<thead>
<tr>
<th>r</th>
<th>p(r)</th>
<th>G(r)</th>
<th>B(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.178</td>
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<td>1.726</td>
</tr>
<tr>
<td>1</td>
<td>0.507</td>
<td>0.685</td>
<td>0.904</td>
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<tr>
<td>2</td>
<td>0.265</td>
<td>0.950</td>
<td>0.389</td>
</tr>
<tr>
<td>3</td>
<td>0.153</td>
<td>0.929</td>
<td>0.140</td>
</tr>
<tr>
<td>4</td>
<td>0.066</td>
<td>0.995</td>
<td>0.042</td>
</tr>
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<td>5</td>
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<td>0.011</td>
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<td>6</td>
<td>0.007</td>
<td>0.998</td>
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<td>0.000</td>
<td>1.000</td>
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</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Calculations for Example

\[ Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(10)(14)}{15}} = 4.3 \approx 4 \]

\[ \frac{b}{h+b} = \frac{40}{25+40} = 0.615 \]

\[ \Phi(0.29) = 0.615, \text{ so } z = 0.29 \]

\[ r^* = \theta + z \sigma = 1.726 + 0.29\sigma = 2.107 \approx 2 \]

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Performance Measures for Example

\[ F(Q^*) = \frac{D}{Q^*} + \frac{1}{2} + 3.5 \]

\[ SQ^* = \int_0^{Q^*} B(r^*)dr^* = \frac{1}{2} [BR^2 + QR^2] - \frac{1}{2} (B(2) - B(2 + 4)] \]

\[ B(Q^*, r^*) = B(Q^*) + B((Q^* + 2) - B(2 + 4) \]

\[ = \frac{1}{2} [0.140 + 0.042 + 0.011 + 0.003] = 0.049 \]

\[ f(Q^*, r^*) = Q^* + r^* - \theta + B(Q^*, r^*) + \frac{3}{2} \] \[ = 2.1726 + 0.049 = 2.282 \]

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Observations on Example

- Orders placed at rate of 3.5 per year
- Fill rate fairly high (90.4%)
- Very few outstanding backorders (0.049 on average)
- Average on-hand inventory just below 3 (2.823)

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Varying the Example

Change: suppose we order twice as often so \( F = 7 \) per year, then \( Q^* = 2 \) and:

\[ S(Q, r) = 1 - \frac{1}{Q} [B(r) - B(r + Q)] = 1 - \frac{1}{2} [0.389 - 0.042] = 0.826 \]

which may be too low, so increase \( r \) from 2 to 3:

\[ S(Q, r) = 1 - \frac{1}{Q} [B(r) - B(r + Q)] = 1 - \frac{1}{2} [0.140 - 0.011] = 0.936 \]

This is better. For this policy (\( Q = 2, r = 4 \)) we can compute \( B(2.3) = 0.026, B(2.3) = 2.80 \).

Conclusion: this has higher service and lower inventory than the original policy (\( Q = 4, r = 2 \)). But the cost of achieving this is an extra 3.5 replenishment orders per year.

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(\( Q, r \)) Model with Stockout Cost

Objective Function:

\[ Y(Q, r) = \frac{D}{Q} A + kD(1 - S(Q, r)) + B(Q, r) \]

Approximation: Assume we can still use EOQ to compute \( Q^* \) but replace \( S(Q, r) \) by Type II approximation and \( B(Q, r) \) by base stock approximation:

\[ Y(Q, r) = \frac{D}{Q} A + kD(1 - S(Q, r)) + B(Q, r) \]

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Results of Approximate Optimization

Assumptions:

- \( Q, r \) can be treated as continuous variables
- \( G(r) \) is a continuous cdf

Results:

\[ Q^* = \sqrt{\frac{2AD}{h}} \]

\[ G(r) = \frac{AD}{kD + hQ} \]

\( Note: this is just the EOQ formula \)

\[ r^* = \theta + z \sigma \]

\( Note: another version of base stock formula (only \( z \) is different) \)

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### Backorder vs. Stockout Model

**Backorder Model**
- When real concern is about stockout time
- Because $B(Q,r)$ is proportional to time orders wait for backorders
- Useful in multi-level systems

**Stockout Model**
- When concern is about fill rate
- Better approximation of lost sales situations (e.g., retail)

**Note:**
- We can use either model to generate frontier of solutions
- Keep track of all performance measures regardless of model
- B-model will work best for backorders, S-model for stockouts

### Lead Time Variability

**Problem:** Replenishment lead times may be variable, which increases variability of lead time demand.

**Notation:**
- $L$ = replenishment lead time (days), a random variable
- $l = E[L]$ = expected replenishment lead time (days)
- $\sigma_L$ = std dev of replenishment lead time (days)
- $D_t$ = demand on day $t$, a random variable assumed independent and identically distributed
- $d = E[D_t]$ = expected daily demand
- $\sigma_D$ = std dev of daily demand (units)

### Including Lead Time Variability in Formulas

**Standard Deviation of Lead Time Demand:**

If demand is Poisson

$$\sigma = \sqrt{l^2 \sigma_D^2 + d^2 \sigma_L^2}$$

Inflation term due to lead time variability

**Modified Base Stock Formula (Poisson demand case):**

$$R = \theta + z \sigma = \theta + \sqrt{l^2 \sigma_D^2 + d^2 \sigma_L^2}$$

Note: $\sigma$ can be used in any base stock or $(Q,s)$ formula as before. In general, it will inflate safety stock.

### Material Requirements Planning (MRP)

Unlike many other approaches and techniques, material requirements planning “works” which is its best recommendation.

— Joseph Orlicky, 1974

### History

- Began around 1960 as computerized approach to purchasing and production scheduling.
- Joseph Orlicky, Oliver Wight, and others.
- APICS launched “MRP Crusade” in 1972 to promote MRP.
Key Insight

• Independent Demand – finished products
• Dependent Demand – components

It makes no sense to independently forecast dependent demands.

Inputs

• Master Production Schedule (MPS): due dates and quantities for all top-level items
• Bills of Material (BOM): for all parent items
• Inventory Status: (on hand plus scheduled receipts) for all items
• Planned Leadtimes: for all items

Assumptions

1. Known deterministic demands.
2. Fixed, known production leadtimes.
3. Infinite capacity.

Idea is to “back out” demand for components by using leadtimes and bills of material.

Example - Stool

Indented BOM

<table>
<thead>
<tr>
<th>Component</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stool</td>
<td>0</td>
</tr>
<tr>
<td>Base (1)</td>
<td>1</td>
</tr>
<tr>
<td>Legs (4)</td>
<td></td>
</tr>
<tr>
<td>Bolts (2)</td>
<td></td>
</tr>
<tr>
<td>Seat (1)</td>
<td></td>
</tr>
<tr>
<td>Bolts (2)</td>
<td></td>
</tr>
</tbody>
</table>

Graphical BOM

<table>
<thead>
<tr>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stool</td>
</tr>
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<td>Base (1)</td>
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<tr>
<td>Legs (4)</td>
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<tr>
<td>Bolts (2)</td>
</tr>
<tr>
<td>Seat (1)</td>
</tr>
<tr>
<td>Bolts (2)</td>
</tr>
</tbody>
</table>

Note: bolts are treated at lowest level in which they occur for MRP calculations. Actually, they might be left off BOM altogether in practice.

MRP Procedure

1. Netting: net requirements against projected inventory
2. Lot Sizing: planned order quantities
3. Time Phasing: planned orders backed out by leadtime
4. BOM Explosion: gross requirements for components

Example

Item: Stool (Leadtime = 1 week)

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td></td>
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<tr>
<td>Proj Inventory</td>
<td>20</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Req</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Item: Base (Leadtime = 1 week)

<table>
<thead>
<tr>
<th>Week</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>Gross Req</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Sched Receipt</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Proj Inventory</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Req</td>
<td>100</td>
<td></td>
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<td></td>
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<tr>
<td>Planned Orders</td>
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<td></td>
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</tr>
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</table>
Example (cont.)

<table>
<thead>
<tr>
<th>Item: Legs (Leadtime = 2 weeks)</th>
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<tr>
<td>Gross Reqs</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>400</td>
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<tr>
<td>Sched Receipts</td>
</tr>
<tr>
<td>200</td>
</tr>
</tbody>
</table>

Lot Sizing in MRP

- Lot-for-lot – “chase” demand
- Fixed order quantity method – constant lot sizes
- EOQ – using average demand
- Fixed order period method – use constant lot intervals
- Part period balancing – try to make setup/ordering cost equal to holding cost
- Wagner-Whitin – “optimal” method

Terminology

- **Level Code**: lowest level on any BOM on which part is found
- **Planning Horizon**: should be longer than longest cumulative leadtime for any product
- **Time Bucket**: units planning horizon is divided into
- **Lot-for-Lot**: batch sizes equal demands (other lot sizing techniques, e.g., EOQ or Wagner-Whitin can be used)
- **Pegging**: identify gross requirements with next level in BOM (single pegging) or customer order (full pegging) that generated it. Single usually used because full is difficult due to lot sizing, yield loss, safety stocks, etc.

Lot Sizing Example

<table>
<thead>
<tr>
<th>t</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<tbody>
<tr>
<td>D</td>
<td>20</td>
<td>50</td>
<td>10</td>
<td>50</td>
<td>50</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>20</td>
<td>30</td>
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<td>130</td>
<td>90</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>20</td>
<td>50</td>
<td>10</td>
<td>50</td>
<td>50</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

\[ W = \frac{1}{2} \left( \frac{D}{A} \right) \]

\[ W = \frac{1}{2} \left( \frac{200}{30} \right) = \frac{100}{30} = 3.33 \]

\[ A = 100 \]

\[ k = 1 \]

\[ S = 300 \]

\[ \text{Wagner-Whitin: } $560 \text{ Note: } WW \text{ is } “\text{optimal}” \text{ given this objective.} \]

Lot-for-Lot: $1000

More Terminology

- **Firm Planned Orders (FPO’s)**: planned order that the MRP system does not automatically change when conditions change – can stabilize system
- **Service Parts**: parts used in service and maintenance – must be included in gross requirements
- **Order Launching**: process of releasing orders to shop or vendors – may include inflation factor to compensate for shrinkage
- **Exception Codes**: codes to identify possible data inaccuracy (e.g., dates beyond planning horizon, exceptionally large or small order quantities, invalid part numbers, etc.) or system diagnostics (e.g., orders open past due, component delays, etc.)
Lot Sizing Example (cont.)

Fixed Order Period (FOP): 3 periods

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Total</th>
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<td>50</td>
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<td>300</td>
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<tr>
<td>C</td>
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<td></td>
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<td></td>
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<td>$620</td>
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</table>

Nervousness Example (cont.)

Item A (Leadtime = 2 weeks, Order Interval = 5 weeks)

<table>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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</thead>
<tbody>
<tr>
<td>Gross</td>
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<td>24</td>
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<td>28</td>
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<td>28</td>
<td>28</td>
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<td>28</td>
</tr>
<tr>
<td>Sched Receipts</td>
<td>2</td>
<td>24</td>
<td>28</td>
<td>28</td>
<td>28</td>
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<td>-6</td>
<td>-6</td>
<td>-6</td>
<td>-6</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>14</td>
<td>14</td>
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<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

Note: we are using FOP lot-sizing rule.

Nervousness Example (cont.)

Item A (Leadtime = 2 weeks, Order Interval = 5 weeks)

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
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<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
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<td>63</td>
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<tr>
<td>Sched Receipts</td>
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<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
</tr>
<tr>
<td>Proj Inventory</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
</tr>
<tr>
<td>Net Reqs</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
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<tr>
<td>Planned Orders</td>
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<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
</tr>
</tbody>
</table>

Note: Small reduction in requirements caused large change in orders and made schedule infeasible.

Handling Change

Causes of Change:
- New orders in MPS
- Order completed late
- Scrap loss
- Engineering changes in BOM

Responses to Change:
- **Regenerative MRP**: completely re-do MRP calculations starting with MPS and exploding through BOMs.
- **Net Change MRP**: store material requirements plan and alter only those parts affected by change (continuously on-line or batched daily).

Comparison:
- Regenerative fixes errors.
- Net change responds faster but must be regenerated periodically.

Rescheduling

Top Down Planning: use MRP system with changes (e.g., altered MPS or scheduled receipts) to recompute plan
- can lead to infeasibilities (exception codes)
- Orlicky suggested using minimum leadtimes
- bottom line is that MPS may be infeasible

Bottom Up Replanning: use pegging and firm planned orders to guide rescheduling process
- pegging allows tracing of release to sources in MPS
- FPO’s allow fixing of releases necessary for firm customer orders
- compressed leadtimes (expediting) are often used to justify using FPO’s to override system leadtimes
Safety Stocks and Safety Leadtimes

Safety Stocks:
- generate net requirements to ensure min level of inventory at all times
- used as hedge against quantity uncertainties (e.g., yield loss)

Safety Leadtimes:
- inflate production leadtimes in part record
- used as hedge against time uncertainty (e.g., delivery delays)

Safety Stock Example

<table>
<thead>
<tr>
<th>Item: Screws (Leadtime = 1 week)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week</td>
</tr>
<tr>
<td>Gross Reqs</td>
</tr>
<tr>
<td>Sched Receipts</td>
</tr>
<tr>
<td>Proj Inventory</td>
</tr>
<tr>
<td>Net Reqs</td>
</tr>
<tr>
<td>Planned Orders</td>
</tr>
</tbody>
</table>

Note: safety stock level is 20.

Safety Stock vs. Safety Leadtime

<table>
<thead>
<tr>
<th>Item: A (Leadtime = 2 weeks, Order Quantity = 50)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week</td>
</tr>
<tr>
<td>Gross Reqs</td>
</tr>
<tr>
<td>Sched Receipts</td>
</tr>
<tr>
<td>Proj Inventory</td>
</tr>
<tr>
<td>Net Reqs</td>
</tr>
<tr>
<td>Planned Orders</td>
</tr>
</tbody>
</table>

Safety Stock vs. Safety Leadtime (cont.)

<table>
<thead>
<tr>
<th>Safety Leadtime = 1 week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week</td>
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<tr>
<td>Gross Reqs</td>
</tr>
<tr>
<td>Sched Receipts</td>
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<td>Proj Inventory</td>
</tr>
<tr>
<td>Net Reqs</td>
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<tr>
<td>Planned Orders</td>
</tr>
</tbody>
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Manufacturing Resource Planning (MRP II)

- Sometime called MRP, in contrast with mrp (“little” mps), more recent implementations are called ERP (Enterprise Resource Planning).
  - Extended MRP into:
    - Master Production Scheduling (MPS)
    - Rough Cut Capacity Planning (RCCP)
    - Capacity Requirements Planning (CRP)
    - Production Activity Control (PAC)

MRP II Planning Hierarchy
Master Production Scheduling (MPS)

- MPS drives MRP
- Should be accurate in near term (firm orders)
- May be inaccurate in long term (forecasts)
- Software supports
  - forecasting
  - order entry
  - netting against inventory
- Frequently establishes a “frozen zone” in MPS

Production Activity Control (PAC)

- Sometimes called “shop floor control”
- Provides routing/standard time information
- Sets planned start times
- Can be used for prioritizing/expediting
- Can perform input-output control (compare planned with actual throughput)
- Modern term is MES (Manufacturing Execution System), which represents functions between Planning and Control.

Rough Cut Capacity Planning (RCCP)

- Quick check on capacity of key resources
- Use Bill of Resource (BOR) for each item in MPS
- Generates usage of resources by exploding MPS against BOR (offset by leadtimes)
- Infeasibilities addressed by altering MPS or adding capacity (e.g., overtime)

Enterprise Resources Planning

Goal: link information across entire enterprise:
- manufacturing
- distribution
- accounting
- financial
- personnel

“Integrated” ERP Approach

Advantages:
- integrated functionality
- consistent user interfaces
- integrated database
- single vendor and contract
- unified architecture
- unified product support

Disadvantages:
- incompatibility with existing systems
- long and expensive implementation
- incompatibility with existing management practices
- loss of flexibility to use tactical point systems
- long product development and implementation cycles
- long payback period
- lack of technological innovation
Other Planning Tools

Manufacturing Execution Systems (MES):
• automated implementation of shop floor control
• data tracking (WIP, yield, quality, etc.)
• merging with ERP?

Advanced Planning Systems (APS):
• algorithms for performing specific functions
• finite capacity scheduling, forecasting, available to promise, demand management, warehouse management, distribution, etc.
• partnerships between developers and ERP vendors

Conclusions

Insight:
distinction between independent and dependent demands

Advantages:
• General approach
• Supports planning hierarchy (MRP II, ERP)

Problems:
• Assumptions – especially infinite capacity
• Cultural factors – e.g., data accuracy, training, etc.
• Focus – authority delegated to computer