Agenda

Web Resources
Schedule
Factory Physics
   Chapter 8: Variability Basics
   Chapter 9: The Corrupting Influence of Variability
   (New Assignment  To be assigned later
   Chapter 8: Problem
   Chapter 9: Problems)
Web Resources

http://sdmines.sdsmt.edu/sdsmt/directory/courses/2009fa/tm663M021-099

I have completed and e-mailed solutions through only September.
## Tentative Schedule

<table>
<thead>
<tr>
<th>Date</th>
<th>Chapters Assigned</th>
<th>Date</th>
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<td>9/28/2009</td>
<td>4, 5 Study Q’s</td>
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Variability Basics

*God does not play dice with the universe.*

– Albert Einstein

*Stop telling God what to do.*

– Niels Bohr
Variability Makes a Difference!

Little’s Law: \( TH = \frac{WIP}{CT} \), so same throughput can be obtained with large WIP, long CT or small WIP, short CT. The difference? \textit{Variability}!

Penny Fab One: achieves full \( TH \) (0.5 j/hr) at \( WIP=W_0=4 \) jobs if it behaves like Best Case, but requires \( WIP=27 \) jobs to achieve 95\% of capacity if it behaves like the Practical Worst Case. Why? \textit{Variability}!
Tortise and Hare Example

Two machines:
- subject to same workload: 69 jobs/day (2.875 jobs/hr)
- subject to unpredictable outages (availability = 75%)

Hare X19:
- long, but infrequent outages

Tortoise 2000:
- short, but more frequent outages

Performance: Hare X19 is substantially worse on all measures than Tortoise 2000. Why?

Variability!
Variability Views

**Variability:**
- Any departure from uniformity
- Random versus controllable variation

**Randomness:**
- Essential reality?
- Artifact of incomplete knowledge?
- Management implications: robustness is key
Probabilistic Intuition

Uses of Intuition:
- driving a car
- throwing a ball
- mastering the stock market

First Moment Effects:
- Throughput increases with machine speed
- Throughput increases with availability
- Inventory increases with lot size
- Our intuition is good for first moments
Probabilistic Intuition (cont.)

Second Moment Effects:

• Which is more variable – processing times of parts or batches?
• Which are more disruptive – long, infrequent failures or short frequent ones?
• Our intuition is less secure for second moments
• Misinterpretation – e.g., regression to the mean
Variability

**Definition**: Variability is anything that causes the system to depart from regular, predictable behavior.

**Sources of Variability**:

- setups
- machine failures
- materials shortages
- yield loss
- rework
- operator unavailability
- workspace variation
- differential skill levels
- engineering change orders
- customer orders
- product differentiation
- material handling
Measuring Process Variability

\[ t_e = \text{mean process time of a job} \]

\[ \sigma_e = \text{standard deviation of process time} \]

\[ c_e = \frac{\sigma_e}{t_e} = \text{coefficient of variation, CV} \]

*Note: we often use the “squared coefficient of variation” (SCV), \( c_e^2 \)*
Variability Classes in Factory Physics®

Effective Process Times:
- *actual* process times are generally LV
- *effective* process times include setups, failure outages, etc.
- HV, LV, and MV are all possible in effective process times

Relation to Performance Cases: For balanced systems
- MV – Practical Worst Case
- LV – between Best Case and Practical Worst Case
- HV – between Practical Worst Case and Worst Case
## Measuring Process Variability – Example

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**Question:** can we measure $c_e$ this way?

**Answer:** No! Won’t consider “rare” events properly.
Natural Variability

**Definition:** variability without explicitly analyzed cause

**Sources:**
- operator pace
- material fluctuations
- product type (if not explicitly considered)
- product quality

**Observation:** natural process variability is usually in the LV category.
Down Time – Mean Effects

Definitions:

\[ t_0 = \text{base process time} \]
\[ c_0 = \text{base process time coefficient of variability} \]
\[ r_0 = \frac{1}{t_0} = \text{base capacity (rate, e.g., parts/hr)} \]
\[ m_f = \text{mean time to failure} \]
\[ m_r = \text{mean time to repair} \]
\[ c_r = \text{coefficient of variability of repair times} \left( \sigma_r / m_r \right) \]
Down Time – Mean Effects (cont.)

**Availability:** Fraction of time machine is up

\[ A = \frac{m_f}{m_f + m_r} \]

**Effective Processing Time and Rate:**

\[ r_e = A r_0 \]

\[ t_e = t_0 / A \]
Totoise and Hare - Availability

Hare X19:
\[ t_0 = 15 \text{ min} \]
\[ \sigma_0 = 3.35 \text{ min} \]
\[ c_0 = \sigma_0 / t_0 = 3.35/15 = 0.05 \]
\[ m_f = 12.4 \text{ hrs (744 min)} \]
\[ m_r = 4.133 \text{ hrs (248 min)} \]
\[ c_r = 1.0 \]

\[ A = \frac{m_f}{m_f + m_r} = \frac{744}{744 + 248} = 0.75 \]

Tortoise:
\[ t_0 = 15 \text{ min} \]
\[ \sigma_0 = 3.35 \text{ min} \]
\[ c_0 = \sigma_0 / t_0 = 3.35/15 = 0.05 \]
\[ m_f = 1.9 \text{ hrs (114 min)} \]
\[ m_r = 0.633 \text{ hrs (38 min)} \]
\[ c_r = 1.0 \]

\[ A = \frac{m_f}{m_f + m_r} = \frac{114}{114 + 38} = 0.75 \]

No difference between machines in terms of availability.
Down Time – Variability Effects

Effective Variability:

\[ t_e = \frac{t_0}{A} \]

\[ \sigma_e^2 = \left( \frac{\sigma_0}{A} \right)^2 + \frac{(m_r^2 + \sigma_r^2)(1 - A)t_0}{Am_r} \]

Conclusions:

\[ c_e^2 = \frac{\sigma_e^2}{t_e^2} = c_0^2 + (1 + c_r^2)A(1 - A)\left(\frac{m_r}{t_0}\right) \]

- Failures inflate mean, variance, and CV of effective process time
- Mean \((t_e)\) increases proportionally with \(1/A\)
- SCV \((c_e^2)\) increases proportionally with \(m_r\)
- SCV \((c_e^2)\) increases proportionally in \(c_r^2\)
- For constant availability \((A)\), long infrequent outages increase SCV more than short frequent ones
Tortoise and Hare - Variability

Hare X19:

\[ t_e = \frac{t_0}{A} = \frac{15}{0.75} = 20 \text{ min} \]

\[ c_e^2 = c_0^2 + (1 + c_r^2)A(1 - A) \frac{m_r}{t_0} = 0.05 + (1 + 1)0.75(1 - 0.75) \frac{248}{15} = 6.25 \text{ high variability} \]

Tortoise 2000:

\[ t_e = \frac{t_0}{A} = \frac{15}{0.75} = 20 \text{ min} \]

\[ c_e^2 = c_0^2 + (1 + c_r^2)A(1 - A) \frac{m_r}{t_0} = 0.05 + (1 + 1)0.75(1 - 0.75) \frac{38}{15} = 1.0 \text{ moderate variability} \]

Hare X19 is much more variable than Tortoise 2000!
Setups – Mean and Variability Effects

Analysis:

\[ N_s = \text{average no. jobs between setups} \]
\[ t_s = \text{average setup duration} \]
\[ \sigma_s = \text{std. dev. of setup time} \]

\[ c_s = \frac{\sigma_s}{t_s} \]

\[ t_e = t_0 + \frac{t_s}{N_s} \]

\[ \sigma_e^2 = \sigma_0^2 + \frac{\sigma_s^2}{N_s} + \frac{N_s - 1}{N_s^2} t_s^2 \]

\[ c_e^2 = \frac{\sigma_e^2}{t_e^2} \]
Setups – Mean and Variability Effects (cont.)

Observations:
- Setups increase mean and variance of processing times.
- Variability reduction is one benefit of flexible machines.
- However, the interaction is complex.
Setup – Example

Data:

- Fast, inflexible machine – 2 hr setup every 10 jobs
  
  \( t_0 = 1 \text{ hr} \)
  
  \( N_s = 10 \text{ jobs/setup} \)
  
  \( t_s = 2 \text{ hrs} \)
  
  \( t_e = t_0 + t_s / N_s = 1 + 2/10 = 1.2 \text{ hrs} \)
  
  \( r_e = 1/t_e = 1/(1+2/10) = 0.8333 \text{ jobs/hr} \)

- Slower, flexible machine – no setups

  \( t_0 = 1.2 \text{ hrs} \)
  
  \( r_e = 1/t_0 = 1/1.2 = 0.833 \text{ jobs/hr} \)

Traditional Analysis?

No difference!
Setup – Example (cont.)

Factory Physics® Approach: Compare mean and variance

- Fast, inflexible machine – 2 hr setup every 10 jobs

\[
t_0 = 1 \text{ hr} \\
c_0^2 = 0.0625 \\
N_s = 10 \text{ jobs/setup} \\
t_s = 2 \text{ hrs} \\
c_s^2 = 0.0625 \\
t_e = t_0 + t_s / N_s = 1 + 2/10 = 1.2 \text{ hrs} \\
r_e = 1/t_e = 1/(1 + 2/10) = 0.8333 \text{ jobs/hr} \\
\sigma_e^2 = \sigma_0^2 + t_s^2 \left( \frac{c_s^2}{N_s} + \frac{N_s - 1}{N_s^2} \right) = 0.4475 \\
c_e^2 = 0.31
\]
Setup – Example (cont.)

- Slower, flexible machine – no setups
  
  \[ t_0 = 1.2 \text{ hrs} \]
  
  \[ c_0^2 = 0.25 \]

  \[ r_e = 1/t_0 = 1/1.2 = 0.833 \text{ jobs/hr} \]
  
  \[ c_e^2 = c_0^2 = 0.25 \]

Conclusion:

*Flexibility can reduce variability.*
Setup – Example (cont.)

**New Machine:** Consider a third machine same as previous machine with setups, but with shorter, more frequent setups

\[ N_s = 5 \text{ jobs/setup} \]
\[ t_s = 1 \text{ hr} \]

**Analysis:**

\[ r_e = 1/t_e = 1/(1+1/5) = 0.833 \text{ jobs/hr} \]

\[ \sigma_e^2 = \sigma_0^2 + t_s^2 \left( \frac{c_s^2}{N_s} + \frac{N_s - 1}{N_s^2} \right) = 0.2350 \]

\[ c_e^2 = 0.16 \]

**Conclusion:**

*Shorter, more frequent setups induce less variability.*
Other Process Variability Inflators

Sources:
• operator unavailability
• recycle
• batching
• material unavailability
• et cetera, et cetera, et cetera

Effects:
• inflate $t_e$
• inflate $c_e$

Consequences:
*Effective process variability can be LV, MV, or HV.*
Illustrating Flow Variability

Low variability arrivals

smooth!

High variability arrivals

bursty!
Measuring Flow Variability

\[ t_a = \text{mean time between arrivals} \]

\[ r_a = \frac{1}{t_a} = \text{arrival rate} \]

\[ \sigma_a = \text{standard deviation of time between arrivals} \]

\[ c_a = \frac{\sigma_a}{t_a} = \text{coefficient of variation of interarrival times} \]
Propagation of Variability

Single Machine Station:

\[ c_d^2(i) = c_a^2(i+1) \]

where \( u \) is the station utilization given by \( u = \frac{r_a t_e}{m} \)

Multi-Machine Station:

\[ c_d^2 = (1 - u^2)(c_a^2 - 1) + \frac{u^2}{m}(c_e^2 - 1) \]

where \( m \) is the number of (identical) machines.
Conclusion: flow variability out of a high utilization station is determined primarily by process variability at that station.
Conclusion: flow variability out of a low utilization station is determined primarily by flow variability into that station.
Variability Interactions

Importance of Queueing:
• manufacturing plants are *queueing networks*
• queueing and waiting time comprise majority of cycle time

System Characteristics:
• Arrival process
• Service process
• Number of servers
• Maximum queue size (blocking)
• Service discipline (FCFS, LCFS, EDD, SPT, etc.)
• Balking
• Routing
• Many more
Kendall's Classification

A/B/C

A: arrival process
B: service process
C: number of machines

M: exponential (Markovian) distribution
G: completely general distribution
D: constant (deterministic) distribution.
Queueing Parameters

\[ r_a = \text{the rate of arrivals in customers (jobs) per unit time} \quad (t_a = 1/r_a = \text{the average time between arrivals}). \]

\[ c_a = \text{the CV of inter-arrival times}. \]

\[ m = \text{the number of machines}. \]

\[ r_e = \text{the rate of the station in jobs per unit time} = m/t_e. \]

\[ c_e = \text{the CV of effective process times}. \]

\[ u = \text{utilization of station} = r_d/r_e. \]

Note: a station can be described with 5 parameters.
Queueing Measures

**Measures:**
- $CT_q = \text{the expected waiting time spent in queue.}$
- $CT = \text{the expected time spent at the process center, i.e., queue time plus process time.}$
- $WIP = \text{the average WIP level (in jobs) at the station.}$
- $WIP_q = \text{the expected WIP (in jobs) in queue.}$

**Relationships:**
- $CT = CT_q + t_e$
- $WIP = r_a \times CT$
- $WIP_q = r_a \times CT_q$

**Result:** If we know $CT_q$, we can compute $WIP$, $WIP_q$, $CT$. 
The G/G/1 Queue

Formula:

\[ CT_q \approx V \times U \times t \]

\[ \approx \left( \frac{c_a^2 + c_e^2}{2} \right) \left( \frac{u}{1-u} \right) t_e \]

Observations:

- Useful model of single machine workstations
- Separate terms for variability, utilization, process time.
- \( CT_q \) (and other measures) increase with \( c_a^2 \) and \( c_e^2 \)
- Flow variability, process variability, or both can combine to inflate queue time.
- Variability causes congestion!
The G/G/m Queue

Formula:

\[ \text{CT}_q \approx V \times U \times t \]
\[ \approx \left( \frac{c_a^2 + c_e^2}{2} \right) \left( \frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)} \right) t_e \]

Observations:

- Useful model of multi-machine workstations
- Extremely general.
- Fast and accurate.
- Easily implemented in a spreadsheet (or packages like MPX).
## VUT Spreadsheet

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<th>STATION:</th>
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<th>3</th>
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<td>Arrival Rate (parts/hr)</td>
<td>$r_s$</td>
<td>10.000</td>
<td>9.800</td>
<td>9.310</td>
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<td>Arrival CV</td>
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<td>$k^2c_0^2/A^2 + 2m_r(1-A)kt_0/A+c_s^2$</td>
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<td>$r_ey$</td>
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<td>7.960</td>
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<td>$yc_d + (1-y)$</td>
<td>0.198</td>
<td>0.079</td>
<td>0.108</td>
<td>0.132</td>
<td>0.077</td>
</tr>
<tr>
<td>Utilization</td>
<td>$u$</td>
<td>0.909</td>
<td>0.940</td>
<td>0.966</td>
<td>0.812</td>
<td>0.731</td>
</tr>
<tr>
<td>Queue Time (hr)</td>
<td>$CT_q$</td>
<td>45.825</td>
<td>14.421</td>
<td>14.065</td>
<td>1.649</td>
<td>0.716</td>
</tr>
<tr>
<td>Cycle Time (hr)</td>
<td>$CT_q + t_e$</td>
<td>54.915</td>
<td>24.011</td>
<td>24.445</td>
<td>10.829</td>
<td>9.896</td>
</tr>
<tr>
<td>Cumulative Cycle Time (hr)</td>
<td>$\Sigma(CT_q(i)+t_e(i))$</td>
<td>54.915</td>
<td>78.925</td>
<td>103.371</td>
<td>114.200</td>
<td>124.096</td>
</tr>
<tr>
<td>WIP in Queue (jobs)</td>
<td>$r_sCT_q$</td>
<td>458.249</td>
<td>141.321</td>
<td>130.948</td>
<td>14.587</td>
<td>5.700</td>
</tr>
<tr>
<td>WIP (jobs)</td>
<td>$r_sCT$</td>
<td>549.149</td>
<td>235.303</td>
<td>227.586</td>
<td>95.780</td>
<td>78.773</td>
</tr>
<tr>
<td>Cumulative WIP (jobs)</td>
<td>$\Sigma(r_s(i)CT(i))$</td>
<td>549.149</td>
<td>784.452</td>
<td>1012.038</td>
<td>1107.818</td>
<td>1186.591</td>
</tr>
</tbody>
</table>

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Effects of Blocking

**VUT Equation:**
- characterizes stations with infinite space for queueing
- useful for seeing what will happen to WIP, CT without restrictions

**But real world systems often constrain WIP:**
- physical constraints (e.g., space or spoilage)
- logical constraints (e.g., kanbans)

**Blocking Models:**
- estimate WIP and TH for given set of rates, buffer sizes
- much more complex than non-blocking (open) models, often require simulation to evaluate realistic systems
The M/M/1/b Queue

Infinite raw materials

1

2

B buffer spaces

Note: there is room for $b=B+2$ jobs in system, $B$ in the buffer and one at each station.

Model of Station 2

$WIP(M/M/1/b) = \frac{u}{1-u} - \frac{(b+1)u^{b+1}}{1-u^{b+1}}$ \hspace{1cm} \text{Goes to } \frac{u}{1-u} \text{ as } b \to \infty \hspace{1cm} \text{Always less than } WIP(M/M/1)$

$TH(M/M/1/b) = \frac{1-u^b}{1-u^{b+1}} r_a$ \hspace{1cm} \text{Goes to } r_a \text{ as } b \to \infty \hspace{1cm} \text{Always less than } TH(M/M/1)$

$CT(M/M/1/b) = \frac{WIP(M/M/1/b)}{TH(M/M/1/b)}$ \hspace{1cm} \text{Little’s law}$

where $u = t_e(2)/t_e(1)$ \hspace{1cm} \text{Note: } u > 1 \text{ is possible; formulas valid for } u \neq 1$
Blocking Example

\[ u = \frac{t_e(2)}{t_e(1)} = \frac{20}{21} = 0.9524 \]

\[ WIP(M/M/1) = \frac{u}{1-u} = 20 \text{ jobs} \]

\[ TH(M/M/1) = r_a = \frac{1}{t_e(1)} = \frac{1}{21} = 0.0476 \text{ job/min} \]

\[ TH(M/M/1/b) = \frac{1-u^b}{1-u^{b+1}} \cdot r_a = \frac{1-0.9524^4}{1-0.9524^5} \left( \frac{1}{21} \right) = 0.039 \text{ job/min} \]

\[ WIP(M/M/1/b) = \frac{u}{1-u} - \frac{(b+1)u^{b+1}}{1-u^{b+1}} = 20 - \frac{5(0.9524^5)}{1-0.9524^5} = 1.8954 \text{ jobs} \]

\[ M/M/1/b \text{ system has less WIP and less TH than M/M/1 system} \]

18% less TH

90% less WIP
Seeking Out Variability

General Strategies:
- look for long queues (Little's law)
- look for blocking
- focus on high utilization resources
- consider both flow and process variability
- ask “why” five times

Specific Targets:
- equipment failures
- setups
- rework
- operator pacing
- anything that prevents regular arrivals and process times
Variability Pooling

**Basic Idea:** the CV of a sum of independent random variables decreases with the number of random variables.

**Example (Time to process a batch of parts):**

\[ t_0 = \text{time to process single part} \]
\[ \sigma_0 = \text{standard deviation of time to process single part} \]
\[ c_0 = \frac{\sigma_0}{t_0} = \text{CV of time to process single part} \]

\[ t_0 (\text{batch}) = nt_0 \]
\[ \sigma_0^2 (\text{batch}) = n\sigma_0^2 \]
\[ c_0^2 (\text{batch}) = \frac{\sigma_0^2 (\text{batch})}{t_0^2 (\text{batch})} = \frac{n\sigma_0^2}{nt_0^2} = \frac{\sigma_0^2}{nt_0^2} = \frac{c_0^2}{n} \implies c_0 (\text{batch}) = \frac{c_0}{\sqrt{n}} \]
Safety Stock Pooling Example

- PC’s consist of 6 components (CPU, HD, CD ROM, RAM, removable storage device, keyboard)
- 3 choices of each component: $3^6 = 729$ different PC’s
- Each component costs $150 ($900 material cost per PC)
- Demand for all models is normally distributed with mean 100 per year, standard deviation 10 per year
- Replenishment lead time is 3 months, so average demand during LT is $\theta = 25$ for computers and $\theta = 25 \times (729/3) = 6075$ for components
- Use base stock policy with fill rate of 99%
Pooling Example - Stock PC’s

Base Stock Level for Each PC:

\[ R = \theta + z_s \sigma = 25 + 2.33(\sqrt{25}) = 37 \]

On-Hand Inventory for Each PC:

\[ I(R) = R - \theta + B(R) \approx R - \theta = z_s \sigma = 37 - 25 = 12 \text{ units} \]

Total (Approximate) On-Hand Inventory:

\[ 12 \times 729 \times \$900 = \$7,873,200 \]
Pooling Example - Stock Components

Necessary Service for Each Component:
\[ S = (0.99)^{1/6} = 0.9983 \quad \Rightarrow \quad z_s = 2.93 \]

Base Stock Level for Each Component:
\[ R = \theta + z_s \sigma = 6075 + 2.93(\sqrt{6075}) = 6303 \]

On-Hand Inventory Level for Each Component:
\[ I(R) = R - \theta + B(R) \approx R - \theta = z_s \sigma = 6303 - 6075 = 228 \text{ units} \]

Total Safety Stock:
\[ 228 \times 18 \times $150 = $615,600 \quad 92\% \text{ reduction!} \]
Basic Variability Takeaways

Variability Measures:
- CV of effective process times
- CV of interarrival times

Components of Process Variability
- failures
- setups
- many others - deflate capacity and inflate variability
- long infrequent disruptions worse than short frequent ones

Consequences of Variability:
- variability causes congestion (i.e., WIP/CT inflation)
- variability propagates
- variability and utilization interact
- pooled variability less destructive than individual variability
The Corrupting Influence of Variability

When luck is on your side, you can do without brains.

— Giordano Bruno, burned at the stake in 1600

The more you know the luckier you get.

— “J.R. Ewing” of Dallas
Performance of a Serial Line

**Measures:**
- Throughput
- Inventory (RMI, WIP, FGI)
- Cycle Time
- Lead Time
- Customer Service
- Quality

**Evaluation:**
- Comparison to “perfect” values (e.g., \( r_b, T_0 \))
- Relative weights consistent with business strategy?

**Links to Business Strategy:**
- Would inventory reduction result in significant cost savings?
- Would CT (or LT) reduction result in significant competitive advantage?
- Would TH increase help generate significantly more revenue?
- Would improved customer service generate business over the long run?

*Remember – standards change over time!*

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**Capacity Laws**

**Capacity Law:** *In steady state, all plants will release work at an average rate that is strictly less than average capacity.*

**Utilization Law:** *If a station increases utilization without making any other change, average WIP and cycle time will increase in a highly nonlinear fashion.*

**Notes:**
- Cannot run at full capacity (including overtime, etc.)
- Failure to recognize this leads to “fire fighting”
Cycle Time vs. Utilization

- High Variability
- Low Variability

Capacity
What Really Happens: System with Insufficient Capacity
What Really Happens:
Two Cases with Releases at 100% of Capacity
What Really Happens:
Two Cases with Releases at 82% of Capacity
Overtime Vicious Cycle

1. Release work at plant capacity.
2. Variability causes WIP to increase.
3. Jobs are late, customers complain,…
4. Authorize one-time use of overtime.
5. WIP falls, cycle times go down, backlog is reduced.
7. Go to Step 1!
Mechanics of Overtime Vicious Cycle

![Graph showing the relationship between cycle time (hrs) and release rate (entities/hr). The graph illustrates the increase in cycle time with the introduction of overtime capacity, highlighting the vicious cycle effect.]
Influence of Variability

**Variability Law:** Increasing variability always degrades the performance of a production system.

**Examples:**

- process time variability pushes best case toward worst case
- higher demand variability requires more safety stock for same level of customer service
- higher cycle time variability requires longer lead time quotes to attain same level of on-time delivery
Variability Buffering

**Buffering Law:** Systems with variability must be buffered by some combination of:

1. inventory
2. capacity
3. time.

**Interpretation:** If you cannot pay to reduce variability, you **will** pay in terms of high WIP, under-utilized capacity, or reduced customer service (i.e., lost sales, long lead times, and/or late deliveries).
Variability Buffering Examples

Ballpoint Pens:
• can’t buffer with time (who will backorder a cheap pen?)
• can’t buffer with capacity (too expensive, and slow)
• must buffer with inventory

Ambulance Service:
• can’t buffer with inventory (stock of emergency services?)
• can’t buffer with time (violates strategic objectives)
• must buffer with capacity

Organ Transplants:
• can’t buffer with WIP (perishable)
• can’t buffer with capacity (ethically anyway)
• must buffer with time
Simulation Studies

**TH Constrained System (push)**

\[ r_a, c_a \quad B(1)=\infty \quad t_e(1), c_e(1) \quad B(2)=\infty \quad t_e(2), c_e(2) \quad B(3)=\infty \quad t_e(3), c_e(3) \quad B(4)=\infty \quad t_e(4), c_e(4) \]

**WIP Constrained System (pull)**

- **Infinite raw materials**

\[ t_e(1), c_e(1) \quad B(2) \quad t_e(2), c_e(2) \quad B(3) \quad t_e(3), c_e(3) \quad B(4) \quad t_e(4), c_e(4) \]

- \( r_a \) = arrival rate
- \( c_a \) = CV of interarrival times
- \( t_e(i) \) = effective process time at station \( i \)
- \( c_e(i) \) = effective CV at station \( i \)
- \( B(i) \) = buffer size in front of station \( i \)
Variability in Push Systems

<table>
<thead>
<tr>
<th>Case</th>
<th>$t_e(i)$, $i = 1, 2, 4$ (min)</th>
<th>$t_e(3)$ (min)</th>
<th>$c(i)$, $i = 1-4$ (unitless)</th>
<th>TH (j/min)</th>
<th>CT (min)</th>
<th>WIP (jobs)</th>
<th>$\sigma_{CT}$ (min)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.2</td>
<td>0</td>
<td>0.8</td>
<td>4.2</td>
<td>3.4</td>
<td>0.0</td>
<td>best case</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.2</td>
<td>1</td>
<td>0.8</td>
<td>44.6</td>
<td>35.7</td>
<td>26.8</td>
<td>WIP buffer</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.0</td>
<td>1</td>
<td>0.8</td>
<td>20.0</td>
<td>16.0</td>
<td>10.3</td>
<td>capacity buffer</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.2</td>
<td>0.3</td>
<td>0.8</td>
<td>7.8</td>
<td>6.2</td>
<td>3.3</td>
<td>reduced variability</td>
</tr>
</tbody>
</table>

Notes:

- $r_a = 0.8$, $c_a = c_e(i)$ in all cases.
- $B(i) = \infty$, $i = 1-4$ in all cases.

Observations:

- TH is set by release rate in a push system.
- Increasing capacity ($r_b$) reduces need for WIP buffering.
- Reducing process variability reduces WIP, CT, and CT variability for a given throughput level.
# Variability in Pull Systems

<table>
<thead>
<tr>
<th>Case</th>
<th>$t_e(i)$, $i = 1, 2, 4$ (min)</th>
<th>$t_e(3)$ (min)</th>
<th>$c(i)$, $i = 1-4$ (unitless)</th>
<th>$B(3)$ (jobs)</th>
<th>TH (j/min)</th>
<th>CT (min)</th>
<th>WIP (jobs)</th>
<th>$\sigma_{CT}$ (min)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.2</td>
<td>0</td>
<td>0</td>
<td>0.83</td>
<td>4.6</td>
<td>3.8</td>
<td>0.0</td>
<td>best case</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.2</td>
<td>1</td>
<td>0</td>
<td>0.48</td>
<td>6.4</td>
<td>3.1</td>
<td>2.4</td>
<td>plain JIT</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.2</td>
<td>1</td>
<td>1</td>
<td>0.53</td>
<td>7.2</td>
<td>3.8</td>
<td>2.6</td>
<td>inv buffer</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.2</td>
<td>0.3</td>
<td>0</td>
<td>0.72</td>
<td>5.0</td>
<td>3.6</td>
<td>0.6</td>
<td>var reduction</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1.2</td>
<td>0.3</td>
<td>1</td>
<td>0.76</td>
<td>6.0</td>
<td>4.5</td>
<td>0.8</td>
<td>inv buffer + var reduction</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1.2</td>
<td>0.3</td>
<td>0</td>
<td>0.73</td>
<td>6.3</td>
<td>4.6</td>
<td>0.7</td>
<td>non-bottleneck buffer</td>
</tr>
</tbody>
</table>

## Notes:
- Station 1 pulls in job whenever it becomes empty.
- $B(i) = 0$, $i = 1, 2, 4$ in all cases, except case 6, which has $B(2) = 1$. 
Variability in Pull Systems (cont.)

Observations:

- Capping WIP without reducing variability reduces TH.
- WIP cap limits effect of process variability on WIP/CT.
- Reducing process variability increases TH, given same buffers.
- Adding buffer space at bottleneck increases TH.
- Magnitude of impact of adding buffers depends on variability.
- Buffering less helpful at non-bottlenecks.
- Reducing process variability reduces CT variability.

Conclusion: consequences of variability are different in push and pull systems, but in either case the buffering law implies that you will pay for variability somehow.
Example – Discrete Parts Flowline

**Inventory Buffers:** raw materials, WIP between processes, FGI

**Capacity Buffers:** overtime, equipment capacity, staffing

**Time Buffers:** frozen zone, time fences, lead time quotes

**Variability Reduction:** smaller WIP & FGI, shorter cycle times
Example – Batch Chemical Process

Inventory Buffers: raw materials, WIP in tanks, finished goods
Capacity Buffers: idle time at reactors
Time Buffers: lead times in supply chain

Variability Reduction: WIP is tightly constrained, so target is primarily throughput improvement, and maybe FGI reduction.
Inventory Buffers: components, in-line buffers
Capacity Buffers: overtime, rework loops, warranty repairs
Time Buffers: lead time quotes

Variability Reduction: initially directed at WIP reduction, but later to achieve better use of capacity (e.g., more throughput)
Buffer Flexibility

**Buffer Flexibility Corollary:** *Flexibility reduces the amount of variability buffering required in a production system.*

**Examples:**
- Flexible Capacity: cross-trained workers
- Flexible Inventory: generic stock (e.g., assemble to order)
- Flexible Time: variable lead time quotes
Variability from Batching

VUT Equation:
- CT depends on process variability \textit{and} flow variability

Batching:
- affects flow variability
- affects waiting inventory

Conclusion: \textit{batching is an important determinant of performance}
Process Batch Versus Move Batch

Dedicated Assembly Line: *What should the batch size be?*

**Process Batch:**
- Related to length of *setup*.
- The longer the setup the larger the lot size required for the same capacity.

**Move (transfer) Batch:** *Why should it equal process batch?*
- The smaller the move batch, the shorter the cycle time.
- The smaller the move batch, the more material handling.

**Lot Splitting:** *Move batch can be different from process batch.*
1. Establish smallest economical move batch.
2. Group batches of like families together at bottleneck to avoid setups.
3. Implement using a “backlog”.

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Process Batching Effects

Types of Process Batching:

1. *Serial Batching*:
   - processes with sequence-dependent setups
   - “batch size” is number of jobs between setups
   - batching used to reduce loss of capacity from setups

2. *Parallel Batching*:
   - true “batch” operations (e.g., heat treat)
   - “batch size” is number of jobs run together
   - batching used to increase effective rate of process
Process Batching

**Process Batching Law:** *In stations with batch operations or significant changeover times:*

1. *The minimum process batch size that yields a stable system may be greater than one.*
2. *As process batch size becomes large, cycle time grows proportionally with batch size.*
3. *Cycle time at the station will be minimized for some process batch size, which may be greater than one.*

**Basic Batching Tradeoff:** WIP versus capacity
Serial Batching

Parameters:

\[ k = \text{serial batch size (10)} \]
\[ t = \text{time to process a single part (1)} \]
\[ s = \text{time to perform a setup (5)} \]
\[ c_e = \text{CV for batch (parts + setup) (0.5)} \]
\[ r_a = \text{arrival rate for parts (0.4)} \]
\[ c_a = \text{CV of batch arrivals (1.0)} \]

**Time to process batch:** \[ t_e = kt + s \]
\[ t_e = 10(1) + 5 = 15 \]
Arrival rate of batches: $r_a/k$

$$r_a = \frac{0.4}{10} = 0.04$$

Utilization: $u = \frac{r_a}{k}(kt + s)$

$$u = 0.04(10 \cdot 1 + 5) = 0.6$$

For stability: $u < 1$ requires

$$k > \frac{sr_a}{1-tr_a}$$

minimum batch size required for stability of system...

$$k > \frac{5(0.4)}{1-1(0.4)} = 3.33$$
Process Batching Effects (cont.)

Average queue time at station:

\[ CT_q = \left( \frac{c_a^2 + c_e^2}{2} \right) \left( \frac{u}{1-u} \right) t_e = \left( \frac{1+0.5}{2} \right) \left( \frac{0.6}{1-0.6} \right) 15 = 16.875 \]

Average cycle time depends on move batch size:

- Move batch = process batch

\[ CT_{non-split} = CT_q + t_e = CT_q + s + kt \]
\[ = 16.875 + 15 = 31.875 \]

- Move batch = 1

\[ CT_{split} = CT_q + s + \frac{k+1}{2} t \]
\[ = 16.875 + 10 + \frac{10+1}{2} (1.0) = 27.375 \]

Note: we assume arrival CV of batches is \( c_a \) regardless of batch size – an approximation...

Note: splitting move batches reduces wait for batch time.
Cycle Time vs. Batch Size – 5 hr setup

Optimum Batch Sizes

Batch Size (jobs/batch) vs. Cycle Time (hrs) for No Lot Splitting and Lot Splitting.
Cycle Time vs. Batch Size – 2.5 hr setup

- No Lot Splitting
- Lot Splitting

Optimum Batch Sizes

Batch Size (jobs/batch)

Cycle Time (hrs)
Setup Time Reduction

Where?
- Stations where capacity is expensive
- Excess capacity may sometimes be cheaper

Steps:
1. Externalize portions of setup
2. Reduce adjustment time (guides, clamps, etc.)
3. Technological advancements (hoists, quick-release, etc.)

Caveat: Don’t count on capacity increase; more flexibility will require more setups.
Parallel Batching

Parameters:

\[ k = \text{parallel batch size} \ (10) \]
\[ t = \text{time to process a batch} \ (90) \]
\[ c_e = \text{CV for batch} \ (1.0) \]
\[ r_a = \text{arrival rate for parts} \ (0.05) \]
\[ c_a = \text{CV of batch arrivals} \ (1.0) \]
\[ B = \text{maximum batch size} \ (100) \]

Time to form batch:

\[ W = \frac{k - 1}{2} \frac{1}{r_a} \]
\[ W = ((10 - 1)/2)(1/0.005) = 90 \]

Time to process batch:

\[ t_e = t \]
\[ t_e = 90 \]
Parallel Batching (cont.)

Arrival of batches: $r_a/k$

$r_a/k = 0.05/10 = 0.005$

Utilization: $u = (r_a/k)(t)$

$u = (0.005)(90) = 0.45$

For stability: $u < 1$ requires $k > r_a t$ minimum batch size required for stability of system...

$k > 0.05(90) = 4.5$
Parallel Batching (cont.)

Average wait-for-batch time:

$$WT = \frac{k - 1}{2} \frac{1}{r_a} = \frac{10 - 1}{2} \frac{1}{0.05} = 90$$

Average queue plus process time at station:

$$CT = \left( \frac{c_a^2}{k + c_0^2} \right) \left( \frac{u}{1-u} \right) t + t = \left( \frac{0.1 + 1}{2} \right) \left( \frac{0.45}{1 - 0.45} \right) 90 + 90 = 130.5$$

Total cycle time:

$$CT + WT = 90 + 130.5 = 220.5$$

batch size affects both wait-for-batch time and queue time
Cycle Time vs. Batch Size in a Parallel Operation

- **Queue time due to utilization**
- **Wait for batch time**
Variable Batch Sizes

**Observation:** Waiting for full batch in parallel batch operation may not make sense. Could just process whatever is there when operation becomes available.

**Example:**
- Furnace has space for 120 wrenches
- Heat treat requires 1 hour
- Demand averages 100 wrenches/hr
- Induction coil can heat treat 1 wrench in 30 seconds
- What is difference between performance of furnace and coil?
Variable Batch Sizes (cont.)

**Furnace:** Ignoring queueing due to variability

- Process starts every hour
- 100 wrenches in furnace
- 50 wrenches waiting on average
- 150 total wrenches in WIP
- $CT = \frac{WIP}{TH} = \frac{150}{100} = \frac{3}{2} \text{ hr} = 90 \text{ min}$

**Induction Coil:** Capacity same as furnace (120 wrenches/hr), but

- $CT = 0.5 \text{ min} = 0.0083 \text{ hr}$
- $WIP = TH \times CT = 100 \times 0.0083 = 0.83 \text{ wrenches}$

**Conclusion:** Dramatic reduction in WIP and CT due to small batches—
independent of variability or other factors.
Move Batching

**Move Batching Law:** *Cycle times over a segment of a routing are roughly proportional to the transfer batch sizes used over that segment, provided there is no waiting for the conveyance device.*

**Insights:**
- Basic Batching Tradeoff: WIP vs. move frequency
- Queueing for conveyance device can offset CT reduction from reduced move batch size
- Move batching intimately related to material handling and layout decisions
Move Batching

Problem:

- Two machines in series
- First machine receives individual parts at rate $r_a$ with CV of $c_a(1)$ and puts out batches of size $k$.
- First machine has mean process time of $t_e(1)$ for one part with CV of $c_e(1)$.
- Second machine receives batches of $k$ and put out individual parts.
- How does cycle time depend on the batch size $k$?
Move Batching Calculations

Time at First Station:

- Average time before batching is:

$$\frac{c_a^2(l) + c_e^2(l)}{2} \frac{u(l)}{1 - u(l)} t_e(1) + t_e(1)$$

- Regular VUT equation...

- Average time forming the batch is:

$$\frac{k - 1}{2} \frac{1}{r_a} = \frac{k - 1}{2u(l)} t_e(1)$$

- First part waits \((k-1)(1/r_a)\), last part doesn’t wait, so average is \((k-1)(1/r_a)/2\)

- Average time spent at the first station is:

$$CT(1) = \frac{c_a^2(l) + c_e^2(l)}{2} \frac{u(l)}{1 - u(l)} t_e(1) + t_e(1) + \frac{k - 1}{2u(l)} t_e(1)$$

$$= CT(1, \text{no batching}) + \frac{k - 1}{2u(l)} t_e(1)$$
Move Batching Calculations (cont.)

Output of First Station:
- Time between output of individual parts into the batch is $t_a$.
- Time between output of batches of size $k$ is $kt_a$.
- Variance of interoutput times of parts is $c_d^2(1)t_a^2$, where
  $$c_d^2(1) = (1 - u(1)^2)c_a^2(1) + u(1)^2 c_e^2(1)$$
- Variance of batches of size $k$ is $kc_d^2(1)t_a^2$.
- SCV of batch arrivals to station 2 is:
  $$c_a^2(2) = \frac{kc_d^2(1)t_a^2}{k^2t_a^2} = \frac{c_d^2(1)}{k}$$
  because departures are independent, so variances add
  because $c_d^2(1) = \sigma_d^2/t_a^2$ by def of CV
  variance divided by mean squared...
Move Batching Calculations (cont.)

Time at Second Station:

- Time to process a batch of size $k$ is $kt_e(2)$.
- Variance of time to process a batch of size $k$ is $kc_e^2(2)t_e^2(2)$.
- SCV for a batch of size $k$ is: 
  \[ \frac{kc_e^2(2)t_e^2(2)}{k^2t_e^2(2)} = \frac{c_e^2(2)}{k} \]
- Mean time spent in partial batch of size $k$ is: 
  \[ \frac{k-1}{2}t_e(2) \]
- So, average time spent at the second station is: 
  \[ \text{CT}(2) = \frac{c_d^2(1)/k + c_e^2(2)/k}{2} + \frac{u(2)}{1-u(2)}kt_e(2) + \frac{k-1}{2}t_e(2) + t_e(2) \]

\[ = \text{CT}(2, \text{no batching}) + \frac{k-1}{2}t_e(2) \]

VUT equation to compute queue time of batches...

independent process times...

first part doesn’t wait, last part waits $(k-1)t_e(2)$, so average is $(k-1)t_e(2)/2$
Move Batching Calculations (cont.)

Total Cycle Time:

\[ CT(\text{batching}) = CT(\text{no batching}) + \frac{k-1}{2u(1)} t_e(1) + \frac{k-1}{2} t_e(2) \]

\[ = CT(\text{no batching}) + \left( \frac{k-1}{2} \right) \left( \frac{t_e(1)}{u(1)} + t_e(2) \right) \]

Insight:

- Cycle time increases with \( k \).
- Inflation term does not involve CV’s
- Congestion from batching is more bad control than randomness.

\textit{inflation factor due to move batching}
**Assembly Operations Law:** The performance of an assembly station is degraded by increasing any of the following:

1. **Number of components being assembled.**
2. **Variability of component arrivals.**
3. **Lack of coordination between component arrivals.**

**Observations:**

- This law can be viewed as a special instance of variability law.
- Number of components affected by product/process design.
- Arrival variability affected by process variability and production control.
- Coordination affected by scheduling and shop floor control.
Attacking Variability

Objectives
- reduce cycle time
- increase throughput
- improve customer service

Levers
- reduce variability directly
- buffer using inventory
- buffer using capacity
- buffer using time
- increase buffer flexibility
Cycle Time

**Definition (Station Cycle Time):** The average cycle time at a station is made up of the following components:

\[ \text{cycle time} = \text{move time} + \text{queue time} + \text{setup time} + \text{process time} + \text{wait-to-batch time} + \text{wait-in-batch time} + \text{wait-to-match time} \]

*delay times typically make up 90% of CT*

**Definition (Line Cycle Time):** The average cycle time in a line is equal to the sum of the cycle times at the individual stations less any time that overlaps two or more stations.
Reducing Queue Delay

\[ CT_q = V \times U \times t \]

\[ \left( \frac{c_a^2 + c_e^2}{2} \right) \left( \frac{u}{1-u} \right) \]

**Reduce Variability**
- failures
- setups
- uneven arrivals, etc.

**Reduce Utilization**
- arrival rate (yield, rework, etc.)
- process rate (speed, time, availability, etc.)
Reducing Batching Delay

\[ CT_{batch} = \text{delay at stations} + \text{delay between stations} \]

**Reduce Process Batching**
- Optimize batch sizes
- Reduce setups
  - Stations where capacity is expensive
  - Capacity vs. WIP/CT tradeoff

**Reduce Move Batching**
- Move more frequently
- Layout to support material handling (e.g., cells)
Reducing Matching Delay

\[ CT_{batch} = \text{delay due to lack of synchronization} \]

**Reduce Variability**
- on high utilization fabrication lines
- usual variability reduction methods

**Improve Coordination**
- scheduling
- pull mechanisms
- modular designs

**Reduce Number of Components**
- product redesign
- kitting
Increasing Throughput

\[ TH = P(\text{bottleneck is busy}) \times \text{bottleneck rate} \]

Reduce Blocking/Starving
- buffer with inventory (near bottleneck)
- reduce system “desire to queue”

\[ CT_q = V \times U \times t \]

Reduce Variability
Reduce Utilization

Increase Capacity
- add equipment
- increase operating time (e.g. spell breaks)
- increase reliability
- reduce yield loss/rework

Note: if WIP is limited, then system degrades via TH loss rather than WIP/CT inflation
Customer Service

Elements of Customer Service:

- lead time
- fill rate (% of orders delivered on-time)
- quality

**Law (Lead Time):** The manufacturing lead time for a routing that yields a given service level is an increasing function of both the mean and standard deviation of the cycle time of the routing.
Improving Customer Service

\[ LT = CT + z \sigma_{CT} \]

**Reduce CT Visible to Customer**
- delayed differentiation
- assemble to order
- stock components

**Reduce Average CT**
- queue time
- batch time
- match time

**Reduce CT Variability**
- generally same as methods for reducing average CT:
  - improve reliability
  - improve maintainability
  - reduce labor variability
  - improve quality
  - improve scheduling, etc.
Cycle Time and Lead Time

Lead Time = 14 days
CT = 10
σ_CT = 3

Lead Time = 27 days
CT = 10
σ_CT = 6
Diagnostics Using Factory Physics®

**Situation:**
- Two machines in series; machine 2 is bottleneck
- $c_a^2 = 1$
- Machine 1: $t_0 = 19\,\text{min}$
  - $c_0^2 = 0.25$
  - MTTF = 48 hr, MTTR = 8 hr
- Machine 2: $t_0 = 22\,\text{min}$
  - $c_0^2 = 1$
  - MTTF = 3.3 hr, MTTR = 10 min
  - Space at machine 2 for 20 jobs of WIP
- Desired throughput 2.4 jobs/hr, not being met
Diagnostic Example (cont.)

Proposal: Install second machine at station 2
  • Expensive
  • Very little space

Analysis Tools:

$$CT_q \approx \frac{c_a^2 + c_e^2}{2} \frac{u}{1-u} t_e$$  \hspace{1cm} VUT equation

$$c_d^2 = u^2 c_e^2 + (1-u^2)c_a^2$$  \hspace{1cm} propagation equation

Analysis:

Step 1: At 2.4 job/hr  \hspace{1cm} *Ask why five times...*
  • $CT_q$ at first station is 645 minutes, average WIP is 25.8 jobs.
  • $CT_q$ at second station is 892 minutes, average WIP is 35.7 jobs.
  • Space requirements at machine 2 are violated!
Diagnostic Example (cont.)

Step 2: Why is \( CT_q \) at machine 2 so big?

- Break \( CT_q \) into

\[
CT_q \approx \left( \frac{c_a^2 + c_e^2}{2} \right) \left( \frac{u}{1-u} \right) t_e = (3.16)(12.22)(23.11 \text{min})
\]

- The 23.11 min term is small.
- The 12.22 correction term is moderate \((u \approx 0.9244)\)
- The 3.16 correction is large.

Step 3: Why is the correction term so large?

- Look at components of correction term.
- \( c_e^2 = 1.04, \ c_a^2 = 5.27 \).
- Arrivals to machine are highly variable.
Diagnostic Example (cont.)

Step 4: Why is $c_a^2$ to machine 2 so large?
- Recall that $c_a^2$ to machine 2 equals $c_d^2$ from machine 1, and
  
  \[ c_d^2 = u^2 c_e^2 + (1 - u^2) c_a^2 = (0.887^2)(6.437) + (1 - 0.887^2)(1.0) = 5.27 \]
- $c_e^2$ at machine 1 is large.

Step 5: Why is $c_e^2$ at machine 1 large?
- Effective CV at machine 1 is affected by failures,

  \[ c_e^2 = c_0^2 + 2A(1 - A) \frac{m_r}{t_0} = 0.25 + 6.18 = 6.43 \]
- The inflation due to failures is large.
- Reducing MTTR at machine 1 would substantially improve performance.
Procoat Case – Situation

Problem:
- Current WIP around 1500 panels
- Desired capacity of 3000 panels/day (19.5 hr day with breaks/lunches)
- Typical output of 1150 panels/day
- Outside vendor being used to make up slack

Proposal:
- Expose is bottleneck, but in clean room
- Expansion would be expensive
- Suggested alternative is to add bake oven for touchups
Procoat Case – Capacity Calculations

<table>
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<tr>
<th>Machine Name</th>
<th>Process or Load Time (min)</th>
<th>Std Dev Process Time (min)</th>
<th>Conveyor Trip Time (min)</th>
<th>Number of Machines</th>
<th>MTTF</th>
<th>MTTR</th>
<th>Avail</th>
<th>Setup Time</th>
<th>Rate (p/day)</th>
<th>Time (min)</th>
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<td>15</td>
<td>1</td>
<td>80</td>
<td>4</td>
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<td>0</td>
<td>15</td>
<td>1</td>
<td>80</td>
<td>4</td>
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</tr>
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</table>

\[ r_b = 2,879 \text{ p/day} \]
\[ T_0 = 546 \text{ min} = 0.47 \text{ days} \]
\[ W_0 = r_b T_0 = 1,343 \text{ panels} \]
Procoat Case – Benchmarking

TH Resulting from PWC with WIP = 1,500:

\[ TH = \frac{w}{w + W_0 - 1} \cdot r_b = \frac{1,500}{1,500 + 1,343 - 1} \cdot 2,879 = 1,520 \]

Higher than actual TH

Conclusion: actual system is significantly worse than PWC.

Question: what to do?
Procoat Case – Factory Physics® Analysis

1) Bottleneck Capacity (Expose)
   - rate: operator training, setup reduction
   - time: break spelling, shift changes

2) Bottleneck Starving
   - process variability: operator training
   - flow variability: coater line – field ready replacements

reduces “desire to queue” so that clean room buffer is adequate
Procoat Case – Outcome

![Graph showing WIP (panels) on the x-axis and TH (panels/day) on the y-axis. The graph displays three lines: Best Case, Practical Worst Case, and Worst Case. Each line is annotated with Before, After, "Good" Region, and "Bad" Region.](image-url)
Corrupting Influence Takeaways

Variance Degrades Performance:
  • many sources of variability
  • planned and unplanned

Variability *Must* be Buffered:
  • inventory
  • capacity
  • time

Flexibility Reduces Need for Buffering:
  • still need buffers, but smaller ones
Corrupting Influence Takeaways (cont.)

Variability and Utilization Interact:
- congestion effects multiply
- utilization effects are highly nonlinear
- importance of bottleneck management

Batching is an Important Source of Variability:
- process and move batching
- serial and parallel batching
- wait-to-batch time in addition to variability effects
Corrupting Influence Takeaways (cont.)

Assembly Operations Magnify Impact of Variability:
  • wait-to-match time
  • caused by lack of synchronization

Variability Propagates:
  • flow variability is as disruptive as process variability
  • non-bottlenecks can be major problems