Chapter 8: Variability Basics

God does not play dice with the universe.

— Albert Einstein

Stop telling God what to do.

— Niels Bohr

Chapter 9: The Corrupting Influence of Variability

Agenda

- Web Resources
- Schedule
- Factory Physics
  - Chapter 8: Variability Basics
  - Chapter 9: The Corrupting Influence of Variability
  - (New Assignment To be assigned later
    - Chapter 8: Problem
    - Chapter 9: Problems)

Tentative Schedule

<table>
<thead>
<tr>
<th>Chapters Assigned</th>
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<tr>
<td>8/31/2009 0,1</td>
<td>11/23/2009 Exam 2</td>
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<td>9/21/2009 2, 3 C3: 2,3,5,6,11</td>
<td>12/14/2009 Final</td>
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<td>9/28/2009 4, 5 Study Q’s 18, 19 Not covered We may</td>
<td>10/5/2009 6, 7 C6:1 C7:5,8,11 rearrange a bit. We could skip</td>
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Variability Makes a Difference!

Little’s Law: TH = WIP/CT, so same throughput can be obtained with large WIP, long CT or small WIP, short CT. The difference? Variability!

Penny Fab One: achieves full TH (0.5 j/hr) at WIP=W_0=4 jobs if it behaves like Best Case, but requires WIP=27 jobs to achieve 95% of capacity if it behaves like the Practical Worst Case. Why? Variability!
**Tortise and Hare Example**

Two machines:
- subject to same workload: 69 jobs/day (2.875 jobs/hr)
- subject to unpredictable outages (availability = 75%)

**Hare X19:**
- long, but infrequent outages

**Tortoise 2000:**
- short, but more frequent outages

**Performance:** Hare X19 is substantially worse on all measures than Tortoise 2000. Why? *Variability!*

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**Variability Views**

**Variability:**
- Any departure from uniformity
- Random versus controllable variation

**Randomness:**
- Essential reality?
- Artifact of incomplete knowledge?
- Management implications: robustness is key

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**Probabilistic Intuition**

**Uses of Intuition:**
- driving a car
- throwing a ball
- mastering the stock market

**First Moment Effects:**
- Throughput increases with machine speed
- Throughput increases with availability
- Inventory increases with lot size
- Our intuition is good for first moments

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**Measuring Process Variability**

\[ t_j = \text{mean process time of a job} \]
\[ \sigma_j = \text{standard deviation of process time} \]
\[ e_j = \frac{\sigma_j}{t_j} = \text{coefficient of variation, CV} \]

*Note: we often use the “squared coefficient of variation” (SCV), \( e_j^2 \)*
Variability Classes in Factory Physics

<table>
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<th>Low variability (LV)</th>
<th>Moderate variability (MV)</th>
<th>High variability (HV)</th>
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Effective Process Times:
- actual process times are generally LV
- effective process times include setups, failure outages, etc.
- HV, LV, and MV are all possible in effective process times

Relation to Performance Cases: For balanced systems
- MV – Practical Worst Case
- LV – between Best Case and Practical Worst Case
- HV – between Practical Worst Case and Worst Case

0.75
High variability (HV)
0
Low variability (LV)
1.33
Moderate variability (MV)

Measuring Process Variability – Example

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c_e

Down Time – Mean Effects

Definitions:
- \( t_0 \): base process time
- \( c_b \): base process time coefficient of variability
- \( c_0 = \frac{1}{c_b} \): base capacity (rate, e.g., parts/hr)
- \( m_f \): mean time to failure
- \( m_r \): mean time to repair
- \( c_r \): coefficient of variability of repair times (\( \sigma_r / m_r \))

Availability:
- \( A = \frac{m_f}{m_f + m_r} \)
- \( t_r = \frac{m_r}{m_f} \)
- \( t_c = t_r / A \)

Tortoise and Hare - Availability

Hare X19:
- \( t_0 = 15 \) min
- \( \sigma_0 = 3.35 \) min
- \( \sigma_0 = 3.35 \times 15 \times 0.05 \)
- \( m_f = 12.4 \) hrs (744 min)
- \( m_r = 1.9 \) hrs (114 min)
- \( c_r = 10 \)

Tortoise:
- \( t_0 = 15 \) min
- \( \sigma_0 = 3.35 \) min
- \( \sigma_0 = 3.35 \times 15 \times 0.05 \)
- \( m_f = 0.63 \) hrs (38 min)
- \( c_r = 10 \)

Availability:
- \( A = \frac{m_r}{m_f + m_r} = \frac{744 + 238}{744 + 238 + 8.75} \)
- \( A = \frac{m_r}{m_f + m_r} = \frac{114 + 38}{114 + 38 + 6.75} \)

No difference between machines in terms of availability.
Fast, inflexible machine setups increase mean variability.

However, the interaction is complex. Variability Effects

SCV (2 hr setup every 10 jobs)

Mean and Variability Effects

\[ \text{Variability} = \text{Mean} \]

\[ r = 22 \]

Mean and Variability Effects (cont.)

For constant availability, slower, flexible machine failures inflate mean, variance, and CV of effective process time.

Example (cont.)

Approach:

Variability

Effective Variability:

\[ t_s - t_s' = t_s - t_s' \]

\[ \sigma_s^2 = \sigma_s'^2 \]

Conclusions: \( c_s^2 = \frac{\sigma_s^2}{\sigma_s'^2} = c_t^2 + (1 + c_t^2) A \]

\[ t_s = t_s' + \frac{t_t}{N_t} \]

\[ \sigma_s^2 = \sigma_t^2 + \frac{N_t - 1}{N_t} \]

\[ c_s^2 = \frac{c_t^2}{c_t'^2} \]

• Failures inflate mean, variance, and CV of effective process time.
• Mean \( t_s' \) increases proportionally with \( A \).
• SCV \( c_t^2 \) increases proportionally with \( r \).
• SCV \( c_s^2 \) increases proportionally with \( c_t^2 \).
• For constant availability \( c_t \), long infrequent outages increase SCV more than short frequent ones.

Setups – Mean and Variability Effects

Analysis:

\[ N_t = \text{average no. jobs between setups} \]

\[ t_s = \text{average setup duration} \]

\[ \sigma_s = \text{std dev of setup time} \]

\[ c_s^2 = \frac{\sigma_s^2}{\sigma_s'^2} \]

\[ t_s = t_s' + t_t / N_t \]

\[ \sigma_s^2 = \sigma_t^2 + \frac{N_t - 1}{N_t} \]

\[ c_s^2 = \frac{c_t^2}{c_t'^2} \]

Setups – Mean and Variability Effects (cont.)

Observations:

• Setups increase mean and variance of processing times.
• Variability reduction is one benefit of flexible machines.
• However, the interaction is complex.

Setup – Example

Data:

• Fast, inflexible machine – 2 hr setup every 10 jobs
  \( t_s = 1 \text{ hr} \)
  \( N_t = 10 \text{ jobs/setup} \)
  \( t_t = 2 \text{ hrs} \)
  \( t_s = t_s' + t_t / N_t = 1 + 2/10 = 1.2 \text{ hrs} \)
• Slower, flexible machine – no setups
  \( t_s = 1.2 \text{ hrs} \)

Traditional Analysis:

\[ \text{No difference!} \]

Setup – Example (cont.)

Factory Physics® Approach: Compare mean and variance

• Fast, inflexible machine – 2 hr setup every 10 jobs
  \( t_s = 1 \text{ hr} \)
  \( c_t^2 = 0.0625 \)
  \( N_t = 10 \text{ jobs/setup} \)
  \( t_t = 2 \text{ hrs} \)
  \( c_t^2 = 0.0625 \)
  \( t_s = t_s' + t_t / N_t = 1 + 2/10 = 1.2 \text{ hrs} \)
  \( t_s = 1/\sqrt{t_s'} + 1(1 + 2/10) = 0.8333 \text{ jobs/hr} \)
  \( \sigma_t^2 = \frac{c_t^2}{c_t'^2} \)
  \[ \sigma_t^2 = 0.0625 \]
  \[ \sigma_s^2 = \frac{c_t^2}{c_t'^2} \]
  \[ \sigma_s^2 = 0.0625 \]
  \[ \sigma_s^2 = 0.0625 \]
  \[ c_t'^2 = 0.4475 \]
  \[ c_t'^2 = 0.31 \]

Hare X19 is much more variable than Tortoise 2000!

For constant availability, much shorter infrequent outages increase SCV more than short frequent ones.
Setup – Example (cont.)

- Slower, flexible machine – no setups
  \[ t_s = 1.2 \text{ hrs} \]
  \[ c_s = 0.25 \]
  \[ c_t = 1/t_s = 1/1.2 = 0.833 \text{ jobs/hr} \]
  \[ c_t^2 = c_s^2 = 0.25 \]

Conclusion: **Flexibility can reduce variability.**

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Setup – Example (cont.)

New Machine: Consider a third machine same as previous machine with setups, but with shorter, more frequent setups

\[ N_s = 5 \text{ jobs/setup} \]

\[ t_s = 1.0 \text{ hr} \]

Analysis:

\[ \sigma_t^2 = \sigma_s^2 + \frac{c_t^2 N_s}{N_s + 1} = 0.2350 \]

\[ c_t^2 = 0.16 \]

Conclusion: **Shorter, more frequent setups induce less variability.**

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Other Process Variability Inflators

**Sources:**
- operator unavailability
- recycle
- batching
- material unavailability
- et cetera, et cetera, et cetera

**Effects:**
- inflate \( t_a \)
- inflate \( c_t \)

**Consequences:**
**Effective process variability can be LV, MV, or HV.**

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Illustrating Flow Variability

- Low variability arrivals: smooth!
- High variability arrivals: bursty!

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Measuring Flow Variability

- \( t_a = \text{mean time between arrivals} \)
- \( r_a = \frac{1}{t_a} = \text{arrival rate} \)
- \( \sigma_{ta} = \text{standard deviation of time between arrivals} \)
- \( c_v = \frac{\sigma_{ta}}{t_a} = \text{coefficient of variation of interarrival times} \)

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Propagation of Variability

**Single Machine Station:**

\[ c_{v(t)} = c_{v(t)} + \frac{c_v^2}{r_a} \]

**Multi-Machine Station:**

\[ c_{v(t)} = 1 + \left( \frac{1}{r_a} \right) u \cdot \left( c_{v(t-1)} - 1 \right) + \frac{c_v^2}{t_a} \]

where \( u \) is the number of identical machines, and \( v(t) \) is the departure variability, \( r_a \) is the arrival variability, and \( c_{v(t)} \) is the process variability.
Propagation of Variability –
High Utilization Station

LV --- HV --- HV
HV --- LV --- HV
LV --- LV --- HV
HV --- LV --- LV

Conclusion: flow variability out of a high utilization station is determined primarily by process variability at that station.

Propagation of Variability –
Low Utilization Station

LV --- HV --- LV
HV --- LV --- HV
LV --- HV --- LV
HV --- HV --- LV

Conclusion: flow variability out of a low utilization station is determined primarily by flow variability into that station.

Variability Interactions

Importance of Queueing:
- manufacturing plants are queueing networks
- queuing and waiting time comprise majority of cycle time

System Characteristics:
- Arrival process
- Service process
- Number of servers
- Maximum queue size (blocking)
- Service discipline (FCFS, LCFS, EDD, SPT, etc.)
- Balking
- Routing
- Many more

Kendall's Classification

A/B/C
A: arrival process
B: service process
C: number of machines
M: exponential (Markovian) distribution
G: completely general distribution
D: constant (deterministic) distribution.

Queueing Parameters

- $r_a$: rate of arrivals in customers (jobs) per unit time ($t_a = 1/r_a$)
- $c_a$: CV of inter-arrival times
- $m$: number of machines
- $r_s$: rate of the station in jobs per unit time
- $c_e$: CV of effective process times
- $u$: utilization of station = $r_s/r_e$

Note: a station can be described with 5 parameters.

Queueing Measures

Measure:
- $CT_q$: expected waiting time spent in queue
- $CT$: expected time spent at process center, i.e., queue time plus process time
- $WIP$: average WIP level (in jobs) at the station
- $WIP_q$: expected WIP (in jobs) in queue

Relationships:
- $CT = CT_q + t_e$
- $WIP = r_a \times CT$
- $WIP_q = r_a \times CT_q$

Result: if we know $CT_q$, we can compute $WIP$, $WIP_q$, $CT$. 

Frank Matejcik SD School of Mines & Technology
Separate terms for variability, utilization, process time.

Flow variability, process variability, or both can combine to inflate queueing time.

Flow variability causes congestion!

Useful model of single machine workstations

Easily implemented in a spreadsheet (or packages like MPX).

Useful model of multi-machine workstations

Extremely general.

Fast and accurate.

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Seeking Out Variability

General Strategies:
• look for long queues (Little's law)
• look for blocking
• focus on high utilization resources
• consider both flow and process variability
• ask “why” five times

Specific Targets:
• equipment failures
• setups
• rework
• operator pacing
• anything that prevents regular arrivals and process times

Variability Pooling

Basic Idea: the CV of a sum of independent random variables decreases with the number of random variables.

Example (Time to process a batch of parts):
\[ t_c = \frac{\sigma_c}{\mu_c} \]
\[ \text{CV of time process single part} \]
\[ \text{CV of time process single part} \]
\[ \text{CV of time process single part} \]

Safety Stock Pooling Example

• PC’s consist of 6 components (CPU, HD, CD ROM, RAM, removable storage device, keyboard)
• 3 choices of each component: 3^6 = 729 different PC’s
• Each component costs $150 ($900 material cost per PC)
• Demand for all models is normally distributed with mean 100 per year, standard deviation 10 per year
• Replenishment lead time is 3 months, so average demand during LT is \( \theta = 25 \) for computers and \( \theta = 25(729/3) = 6075 \) for components
• Use base stock policy with fill rate of 99%

Basic Variability Takeaways

Variability Measures:
• CV of effective process times
• CV of interarrival times

Components of Process Variability
• failures
• setups
• many others - deflate capacity and inflate variability
• long infrequent disruptions worse than short frequent ones

Consequences of Variability:
• variability causes congestion (i.e., WIP/CT inflation)
• variability propagates
• variability and utilization interact
• pooled variability less destructive than individual variability

Pooling Example - Stock PC’s

Base Stock Level for Each PC:
\[ R = \theta + z_s \sigma = 25 + 2.33(\sqrt{25}) = 37 \]

On-Hand Inventory for Each PC:
\[ I(R) = R - \theta + B(R) \approx R - \theta = z_s \sigma = 37 - 25 = 12 \text{ units} \]

Total (Approximate) On-Hand Inventory:
\[ 12 \times 729 \times 900 = 7,873,200 \]

Pooling Example - Stock Components

Necessary Service for Each Component:
\[ S = (0.99)^{\theta} = 0.9936 \Rightarrow z_s = 2.93 \]

Base Stock Level for Each Component: \( R = \theta + z_s \sigma = 6075 + 2.93(\sqrt{6075}) = 6303 \)

On-Hand Inventory Level for Each Component:
\[ B(R) = R - \theta + B(R) \Rightarrow R - \theta = z_s \sigma = 6303 - 6075 = 228 \text{ units} \]

Total Safety Stock:
\[ 228 \times 18 \times 150 = 615,600 \text{ 92% reduction!} \]
The Corrupting Influence of Variability

When luck is on your side, you can do without brains.

– Giordano Bruno, burned at the stake in 1600

The more you know the luckier you get.

– “J.R. Ewing” of Dallas

Performance of a Serial Line

Measures:
- Throughput
- Inventory (RMI, WIP, FGI)
- Cycle Time
- Lead Time
- Customer Service
- Quality

Evaluation:
- Comparison to “perfect” values (e.g., $c_t, T_0$)
- Relative weights consistent with business strategy?

Links to Business Strategy:
- Would inventory reduction result in significant cost savings?
- Would CT (or LT) reduction result in significant competitive advantage?
- Would TH increase help generate significantly more revenue?
- Would improved customer service generate business over the long run?

Remember – standards change over time!

Capacity Laws

Capacity Law: In steady state, all plants will release work at an average rate that is strictly less than average capacity.

Utilization Law: If a station increases utilization without making any other change, average WIP and cycle time will increase in a highly nonlinear fashion.

Notes:
- Cannot run at full capacity (including overtime, etc.)
- Failure to recognize this leads to “fire fighting”

Cycle Time vs. Utilization

What Really Happens:
System with Insufficient Capacity

What Really Happens:
Two Cases with Releases at 100% of Capacity
What Really Happens: Two Cases with Releases at 82% of Capacity

Overtime Vicious Cycle

1. Release work at plant capacity.
2. Variability causes WIP to increase.
3. Jobs are late, customers complain....
4. Authorize one-time use of overtime.
5. WIP falls, cycle times go down, backlog is reduced.
7. Go to Step 1!

Mechanics of Overtime Vicious Cycle

Influence of Variability

Variability Law: Increasing variability always degrades the performance of a production system.

Examples:
- Process time variability pushes best case toward worst case
- Higher demand variability requires more safety stock for same level of customer service
- Higher cycle time variability requires longer lead time quotes to attain same level of on-time delivery

Variability Buffering

Buffering Law: Systems with variability must be buffered by some combination of:
1. inventory
2. capacity
3. time.

Interpretation: If you cannot pay to reduce variability, you will pay in terms of high WIP, under-utilized capacity, or reduced customer service (i.e., lost sales, long lead times, and/or late deliveries).

Variability Buffering Examples

Ballpoint Pens:
- Can’t buffer with time (who will backorder a cheap pen?)
- Can’t buffer with capacity (too expensive, and slow)
- Must buffer with inventory

Ambulance Service:
- Can’t buffer with inventory (stock of emergency services?)
- Can’t buffer with time (violates strategic objectives)
- Must buffer with capacity

Organ Transplants:
- Can’t buffer with WIP (perishable)
- Can’t buffer with capacity (ethically anyway)
- Must buffer with time
Adding buffer space at bottleneck increases TH,
smaller WIP & FGI, shorter cycle times

Reducing process variability increases TH, given same buffers.

\[ t_e = 0.8 \]

lead times in supply chain

Buffering less helpful at non-bottleneck
Batch Chemical Process

WIP is tightly constrained, so target is primarily throughput improvement, and maybe FGI reduction.

Notes:
- \( c_r = 0.8 \), \( c_i = 1 \) in all cases.
- \( B(i) = \infty \), \( r(i) = 1 \) in all cases.

Observations:
- TH is set by release rate in a push system.
- Increasing capacity reduces WIP, CT, and CT variability.
- Reducing process variability reduces WIP, CT, and CT variability for a given throughput level.

Conclusion: consequences of variability are different in push and pull systems, but in either case the buffering law implies that you will pay for variability somehow.

Notes:
- Station 1 pulls in job whenever it becomes empty.
- \( B(i) = \infty \), \( r(i) = 1 \) in all cases, except case 6, which has \( B(2) = 1 \).
Example – Moving Assembly Line

- Inventory Buffers: components, in-line buffers
- Capacity Buffers: overtime, rework loops, warranty repairs
- Time Buffers: lead time quotes
- Variability Reduction: initially directed at WIP reduction, but later to achieve better use of capacity (e.g., more throughput)

Buffer Flexibility

- Buffer Flexibility Corollary: Flexibility reduces the amount of variability buffering required in a production system.
- Examples:
  - Flexible Capacity: cross-trained workers
  - Flexible Inventory: generic stock (e.g., assemble to order)
  - Flexible Time: variable lead time quotes

Variability from Batching

- VUT Equation:
  - CT depends on process variability and flow variability
- Batching:
  - affects flow variability
  - affects waiting inventory
- Conclusion: batching is an important determinant of performance

Process Batch Versus Move Batch

- Dedicated Assembly Line: What should the batch size be?
- Process Batch:
  - Related to length of setup
  - The longer the setup the larger the lot size required for the same capacity
- Move (transfer) Batch: Why should it equal process batch?
  - The smaller the move batch, the shorter the cycle time
  - The smaller the move batch, the more material handling
- Lot Splitting: Move batch can be different from process batch
  1. Establish smallest economical move batch
  2. Group batches of like families together at bottleneck to avoid setups
  3. Implement using a “backlog”

Process Batching

- Process Batching Law: In stations with batch operations or significant changeover times:
  1. The minimum process batch size that yields a stable system may be greater than one.
  2. As process batch size becomes large, cycle time grows proportionally with batch size.
  3. Cycle time at the station will be minimized for some process batch size, which may be greater than one.
- Basic Batching Tradeoff: WIP versus capacity

Types of Process Batching:

1. Serial Batching:
   - processes with sequence-dependent setups
   - “batch size” is number of jobs between setups
   - batching used to reduce loss of capacity from setups
2. Parallel Batching:
   - true “batch” operations (e.g., heat treat)
   - “batch size” is number of jobs run together
   - batching used to increase effective rate of process
Serial Batching

Parameters:

- $k =$ serial batch size (10)
- $t =$ time to process a single part (1)
- $s =$ time to perform setup (5)
- $c_r =$ CV for batch (parts + setup) (0.5)
- $r_e =$ arrival rate for parts (0.4)
- $c_e =$ CV of batch arrivals (1.0)

Time to process batch: $t_e = kt + s$

$\frac{r_e}{s} \frac{c_r}{c_e}$

Arrival rate of batches: $r_a$

Utilization: $u = \frac{r_a}{k(t + s)}$

Cycle Time vs. Batch Size

Cycle Time vs. Batch Size – 2.5 hr setup

Cycle Time vs. Batch Size – 5 hr setup

Process Batching Effects (cont.)

Average queue time at station:

$CT_q = \frac{\frac{k}{2} + \frac{s}{2}}{1 - \frac{k + 0.5}{2}} \cdot \left(1 - \frac{0.6}{1 - 0.6}\right)$

$= 15 \cdot 16.875$

Average cycle time for move batch size:

- Moving batch = process batch

$CT_{mov} = CT_e + \frac{c_r}{c_e}$

- Moving batch = 1

$CT_{mov} = CT_e + \frac{k + \frac{1}{k}}{2}$

Note: splitting move batches reduces wait for batch time.

Note: we assume arrival CV of batches is $c_r$ regardless of batch size – an approximation.

Setup Time Reduction

Where?

- Stations where capacity is expensive
- Excess capacity may sometimes be cheaper

Steps:

1. Externalize portions of setup
2. Reduce adjustment time (guides, clamps, etc.)
3. Technological advancements (hoists, quick-release, etc.)

Caveat: Don’t count on capacity increase; more flexibility will require more setups.
Parallel Batching

Parameters:
- $k = \text{parallel batch size} (10)$
- $t = \text{time to process a batch} (90)$
- $c_r = \text{CV for batch} (1.0)$
- $c_a = \text{arrival rate for parts} (0.05)$
- $c_v = \text{CV of batch arrivals} (1.0)$
- $B = \text{maximum batch size} (100)$

Time to form batch:
$$W = \frac{k-1}{2} \frac{1}{c_r} \frac{r_a}{k} \frac{c_a}{(1-u)} \approx 90$$

Time to process batch:
$$t = \frac{100}{c_v}$$

Parallel Batching (cont.)

Arrival of batches:
$$r_a/k$$

Utilization:
$$u = \frac{r_a/k}{b}$$

For stability: $u < 1$ requires $k > 0.05/10 = 0.005$

Variable Batch Sizes

Observation: Waiting for full batch in parallel batch operation may not make sense. Could just process whatever is there when operation becomes available.

Example:
- Furnace has space for 120 wrenches
- Heat treat requires 1 hour
- Demand averages 100 wrenches/hr
- Induction coil can heat treat 1 wrench in 30 seconds
- What is difference between performance of furnace and coil?

Cycle Time vs. Batch Size in a Parallel Operation

Variable Batch Sizes (cont.)

Furnace:
- Ignoring queueing due to variability
- Process starts every hour
- 100 wrenches in furnace
- 50 wrenches waiting on average
- 150 total wrenches in WIP
- $CT = WIP/TH = 150/100 = 3/2 \text{ hr} = 90 \text{ min}$

Induction Coil:
- Capacity same as furnace (120 wrenches/hr), but
- $CT = 0.5 \text{ min} = 0.0083 \text{ hr}$
- $WIP = TH \times CT = 100 \times 0.0083 = 0.83$ wrenches

Conclusion: Dramatic reduction in WIP and CT due to small batches—
independent of variability or other factors.
Move Batching

**Move Batching Law:** Cycle times over a segment of a routing are roughly proportional to the transfer batch sizes used over that segment, provided there is no waiting for the conveyance device.

**Insights:**
- Basic Batching Tradeoff: WIP vs. move frequency
- Queuing for conveyance device can offset CT reduction from reduced move batch size
- Move batching intimately related to material handling and layout decisions

How does cycle time depend on the batch size?

**Basic Batching Tradeoff:**  
WIP vs. move frequency

**Queueing for conveyance device** can offset CT reduction from reduced move batch size.

Congestion from batching is more bad control than randomness.

**Move Batching Calculations (cont.)**

**Output of First Station:**
- Time between output of individual parts into the batch is $\tau_k$.
- Time between output of batches of size $k$ is $\lambda_k$.
- Variance of interoutput times of parts is $\tau^2(1)$, with $\tau^2(1)=\tau^2(1)+\tau^2(2)$
- Variance of batches of size $k$ is $\lambda_k\tau^2(1)$
- SCV of batch arrivals to station 2 is $\frac{\lambda_k\tau^2(1)}{\lambda_k\tau^2(1)}$ because departures are independent, so variances add

**Variance divided by mean squared:**

$\frac{\lambda_k\tau^2(1)}{\lambda_k\tau^2(1)} = \frac{\tau^2(2)}{\tau^2(2)}$

**Move Batching Calculations (cont.)**

**Time at Second Station:**
- Time to process a batch of size $k$ is $k\tau(2)$
- Variance of time to process a batch of size $k$ is $k\tau^2(2)$
- SCV for a batch of size $k$ is $\frac{k\tau^2(2)}{k\tau^2(2)}$
- Mean time spent in partial batches of size $k$ is $\frac{k-1}{k}\tau(2)$
- So, average time spent at the second station is:

$$CT(2) = \frac{\tau^2(1)/k + \tau^2(2)/k \cdot \frac{k-1}{k} \tau(2)}{1-\frac{k-1}{k}} = \frac{\tau^2(1)/k + \tau^2(2)/k \cdot \frac{k-1}{k} \tau(2)}{1-\frac{k-1}{k}}$$

**Total Cycle Time:**

$$CT(batching) = CT(no\ batching) + \frac{k-1}{2\tau(2)} + \frac{k-1}{2\tau(2)}$$

**Insight:**
- Cycle time increases with $k$.
- Inflation term does not involve CV’s.
- Congestion from batching is more bad control than randomness.
Assembly Operations

Assembly Operations Law: The performance of an assembly station is degraded by increasing any of the following:
1. Number of components being assembled.
2. Variability of component arrivals.
3. Lack of coordination between component arrivals.

Observations:
• This law can be viewed as a special instance of variability law.
• Number of components affected by product/process design.
• Arrival variability affected by process variability and production control.
• Coordination affected by scheduling and shop floor control.

Attacking Variability

Objectives
• reduce cycle time
• increase throughput
• improve customer service

Levers
• reduce variability directly
  • buffer using inventory
  • buffer using capacity
  • buffer using time
  • increase buffer flexibility

Cycle Time

Definition (Station Cycle Time): The average cycle time at a station is made up of the following components:
\[ CT_s = \text{move time} + \text{queue time} + \text{setup time} + \text{process time} + \text{wait-to-batch time} + \text{wait-in-batch time} + \text{wait-to-match time} \]

Definition (Line Cycle Time): The average cycle time in a line is equal to the sum of the cycle times at the individual stations less any time that overlaps two or more stations.

Reducing Queue Delay

\[ CT_q = V \times U \times t \]

\[ \left( \frac{c_1^2 + c_2^2}{2} \right) \left( \frac{a}{1-u} \right) \]

Reduce Variability
• failures
• setups
• uneven arrivals, etc.

Reduce Utilization
• arrival rate (yield, rework, etc.)
• process rate (speed, time, availability, etc.)

Reducing Batching Delay

\[ CT_{\text{batch}} = \text{delay at stations} + \text{delay between stations} \]

Reduce Process Batching
• Optimize batch sizes
• Reduce setups
  – Stations where capacity is expensive
  – Capacity vs. WIP/CT tradeoff

Reduce Move Batching
• Move more frequently
• Layout to support material handling (e.g., cells)

Reducing Matching Delay

\[ CT_{\text{match}} = \text{delay due to lack of synchronization} \]

Reduce Variability
• on high utilization fabrication lines
• usual variability reduction methods

Improve Coordination
• scheduling
• pull mechanisms
• modular designs

Reduce Number of Components
• product redesign
• kitting
**Increasing Throughput**

\[ TH = P(\text{bottleneck is busy}) \times \text{bottleneck rate} \]

**Reduce Blocking/Starving**
- buffer with inventory (near bottleneck)
- reduce system “desire to queue”

**Increase Capacity**
- add equipment
- increase operating time (e.g. spell breaks)
- increase reliability
- reduce yield loss/rework

**Reduce Variability**
- Reduce Utilization

**Note**: if WIP is limited, then system degrades via TH loss rather than WIP/CT inflation

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**Customer Service**

**Elements of Customer Service:**
- lead time
- fill rate (% of orders delivered on-time)
- quality

**Law (Lead Time):** The manufacturing lead time for a routing that yields a given service level is an increasing function of both the mean and standard deviation of the cycle time of the routing.

---

**Improving Customer Service**

\[ LT = CT + z \sigma_{CT} \]

**Reduce Average CT**
- queue time
- batch time
- match time

**Reduce CT Variability**
- generally same as methods for reducing average CT:
  - improve reliability
  - improve maintainability
  - reduce labor variability
  - improve quality
  - improve scheduling, etc.

---

**Cycle Time and Lead Time**

**Diagnostics Using Factory Physics®**

**Situation:**
- Two machines in series; machine 2 is bottleneck
- \( c_2 = 1 \)
- Machine 1:  \( t_a = 19 \text{ min} \)
  \( c_1^2 = 0.25 \)
  MTTF = 48 hr, MTTR = 8 hr
- Machine 2:  \( t_a = 22 \text{ min} \)
  \( c_2^2 = 1 \)
  MTTF = 3.3 hr, MTTR = 10 min

  - Space at machine 2 for 20 jobs of WIP
  - Desired throughput 2.4 jobs/hr, not being met

**Proposal:** Install second machine at station 2
- Expensive
- Very little space

**Analysis Tools:**
- VUT equation
- \( c_{T'} = \frac{c_1^2 + c_2^2}{2} \times \frac{1}{1-c_1 c_2} \)
- propagation equation

**Analysis:**
- **Step 1:** At 2.4 jobs/hr
  - \( C_{T'} \) at first station is 645 minutes, average WIP is 25.8 jobs.
  - \( C_{T'} \) at second station is 892 minutes, average WIP is 35.7 jobs.
  - Space requirements at machine 2 are violated!

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**Diagnostic Example (cont.)**

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Diagnostic Example (cont.)

Step 2: Why is CT, at machine 2 so big?
- Break CT into
  \[ CT = \left( \frac{c_1 + c_2}{2} \right) \frac{t}{1 - w} = (3.16)(2.22)(23.11) \]
- The 23.11 min term is small.
- The 3.16 correction term is moderate (\( u = 0.9244 \)).

Step 3: Why is the correction term so large?
- Look at components of correction term.
  - \( c_2 = 1.06 \), \( c_1 = 5.27 \).
  - Arrivals to machine are highly variable.

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Procoat Case – Situation

Problem:
- Current WIP around 1500 panels
- Desired capacity of 3000 panels/day (19.5 hr day with breaks/lunches)
- Typical output of 1150 panels/day
- Outside vendor being used to make up slack

Proposal:
- Expose is bottleneck, but in clean room
- Expansion would be expensive
- Suggested alternative is to add bake oven for touchups

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Procoat Case – Capacity Calculations

<table>
<thead>
<tr>
<th>Machine</th>
<th>Processing Time (min)</th>
<th>Setup Time (min)</th>
<th>Rate (p/day)</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coat1</td>
<td>0.33</td>
<td>0</td>
<td>3277</td>
<td>38.3</td>
</tr>
<tr>
<td>Coat2</td>
<td>0.33</td>
<td>0.5</td>
<td>3277</td>
<td>38.3</td>
</tr>
<tr>
<td>Expose</td>
<td>0.67</td>
<td>0</td>
<td>3679</td>
<td>43.7</td>
</tr>
<tr>
<td>Washing</td>
<td>0.33</td>
<td>0.5</td>
<td>3679</td>
<td>43.7</td>
</tr>
<tr>
<td>Inspect</td>
<td>0.5</td>
<td>0.5</td>
<td>4580</td>
<td>5.0</td>
</tr>
<tr>
<td>Bake</td>
<td>0.5</td>
<td>0.5</td>
<td>4580</td>
<td>5.0</td>
</tr>
<tr>
<td>WIP</td>
<td>1.5</td>
<td>0</td>
<td>2510</td>
<td>29.7</td>
</tr>
<tr>
<td>Total</td>
<td>14.5</td>
<td>14.5</td>
<td>2879</td>
<td>34.8</td>
</tr>
</tbody>
</table>

\( r_T = 2.879 \text{ p/day} \)
\( T_T = 546 \text{ min} = 0.47 \text{ days} \)
\( W_T = r_T T_T = 1,343 \text{ panels} \)

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Procoat Case – Layout

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Procoat Case – Benchmarking

TH Resulting from PWC with WIP = 1,500:
\[
TH = \frac{w}{w + W_T} T_T = \frac{1,500}{1,500 + 1,343} = 0.520
\]
\( 2.879 = 1.520 \text{ Higher than actual TH} \)

Conclusion: actual system is significantly worse than PWC.

Question: what to do?

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Procoat Case – Factory Physics® Analysis

1) Bottleneck Capacity (Exposé)
   - rate: operator training, setup reduction
   - time: break spelling, shift changes

2) Bottleneck Starving
   - process variability: operator training
   - flow variability: coater line – field ready replacements

reduces “desire to queue” so that clean room buffer is adequate

Procoat Case – Outcome

Corrupting Influence Takeaways

Variance Degrades Performance:
• many sources of variability
• planned and unplanned

Variability Must be Buffered:
• inventory
• capacity
• time

Flexibility Reduces Need for Buffering:
• still need buffers, but smaller ones

Corrupting Influence Takeaways (cont.)

Variability and Utilization Interact:
• congestion effects multiply
• utilization effects are highly nonlinear
• importance of bottleneck management

Batching is an Important Source of Variability:
• process and move batching
• serial and parallel batching
• wait-to-batch time in addition to variability effects

Corrupting Influence Takeaways (cont.)

Assembly Operations Magnify Impact of Variability:
• wait-to-match time
• caused by lack of synchronization

Variability Propagates:
• flow variability is as disruptive as process variability
• non-bottlenecks can be major problems