Problem:

Design a PD Controller \( K(s + a) \) such that the closed loop poles are located at \( \mathbf{s} = -25 \pm 2i \). Calculate what you would expect from a second order system with poles at this location for settling time, time to peak, and \( T_p \). Simulate the compensated system and determine the actual values for settling time, time to peak, and \( T_p \).

Design Solution:

\[ \theta_c = 180 - \tan^{-1}\left(\frac{1}{2}\right) = 135^\circ \]
\[ \theta_p = 90^\circ \]
\[ 350 = \theta_c - \left[ 90 + 135 \right] \Rightarrow 180 + 225 = \theta_c \]
\[ \theta_c = 45^\circ \]

The pole at \( s = -25 \pm 2i \) would imply:

- \( T_s = 0.5 \) sec
- \( T_p = 1.57 \) sec

Compensated System:

- \( T_p = 0.77 \) sec
- \( T_s = 1.54 \) sec

\[ K_c = \frac{\sqrt{2^2 + 2^2}}{\sqrt{(4-2)^2 + 3^2}} \]
\[ G_c = \frac{2}{(s+2)(s+3)} \]
Step Response

- Peak amplitude: 1.07
- Overshoot (%): 6.7
- At time (sec): 1.17
- Setting Time (sec): 1.86

Problem 1 - AP103 Compensator
Problem 2

Given: \( G_p(s) = \frac{s^2 + 2s + 1}{s^2 + 2s + 4} \)
\( R(s) = \frac{3}{s} \)

Design: A lead compensator
\( G_c(s) = \frac{K(s + \theta)}{(s + \phi)(s + \psi)} \) to improve \( T(s) \) by a factor of 2 while keeping \( L = 0.5 \)
Simulate results

Before Compensation
\( T(s) = \frac{G_p(s)}{1 + G_p(s)} \)
\( T(s) = \frac{3}{s^2 + 2s + 4} \)

Poles: \(-1 \pm j1.73\)
\( T_c = \frac{4}{\pi} > \frac{2}{\pi} = 4.28 \) sec

Desired Pole Location
Real Part Needs To Be
At -2 \( (T_c = \frac{4}{\pi} = 2.28 \) sec\)

\( \tan^{-1}(\frac{\omega_c}{\zeta}) = 60^\circ \)
\( \omega_c = 3.46 \)

Desired OL Pole: \(-2 + j3.46\)

Using Magnitude Criterion
\( K = \frac{\sqrt{3.46^2 + 3.46^2}}{\sqrt{2^2 + 3.46^2}} \)
\( K = 3.42 \)
\( K = K_c K_p \Rightarrow 3.42 = K_c (3) \)
\( K_c = 0.8 \)

Using Angle Criterion
\( \Theta_{pc} = \Theta_{pc} = \Theta_{pc} \)
\( \Theta_{pc} = 106.1^\circ \)
\( \Theta_{pc} = \tan^{-1}(\frac{3.46}{2.46}) = 60^\circ \)
\( \Theta_{pc} = 60 \tan^{-1}(\frac{3.46}{2.46}) + 180^\circ \)
\( \Theta_{pc} = 27.7^\circ \)
\( \tan(27.7) = \frac{3.46}{2.46} \)
\( \phi_c = \tan^{-1}(\frac{3.46}{2.46}) = 51.7^\circ \)
\( \phi_c = 0.9 \tan^{-1}(\frac{3.46}{2.46}) = 8.6 \)

\( G_c(s) = \frac{8.06(s + 3)}{5(s + 6)} \)
Problem 2 - BEFORE COMPENSATOR

System: sys1
Peak amplitude: 0.872
Overshoot (\%): 16.3
At time (sec): 1.9

System: sys1
Settling Time (sec): 4.04

Problem 2 - AFTER COMPENSATOR

System: sys2
Peak amplitude: 1.14
Overshoot (\%): 23.5
At time (sec): 0.704

System: sys2
Settling Time (sec): 2.01

NOTE MORE Overshoot caused by zero - IF important, may redesign with higher zeros or choose different zero location.