I have most of your exams graded.
Expect to have the study guide post and solutions sent out tonight.
Agenda

Factory Physics

Chapter 16: Aggregate and Workforce Planning
Chapter 17: Supply Chain Management

(New Assignment
Chapter 16 problems 1-4
Chapter 17 problem 1)
# Tentative Schedule

<table>
<thead>
<tr>
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Aggregate Planning

And I remember misinformation followed us like a plague,
Nobody knew from time to time if the plans were changed.

– Paul Simon
Aggregate Planning Issues

Role of Aggregate Planning
- Long-term planning function
- Strategic preparation for tactical actions

Aggregate Planning Issues
- *Production Smoothing*: inventory build-ahead
- *Product Mix Planning*: best use of resources
- *Staffing*: hiring, firing, training
- *Procurement*: supplier contracts for materials, components
- *Sub-Contracting*: capacity vending
- *Marketing*: promotional activities
Hierarchical Production Planning

- **Marketing Parameters**
- **Product/Process Parameters**
- **FORECASTING**
  - **CAPACITY/FACILITY PLANNING**
  - **WORKFORCE PLANNING**
    - **Personnel Plan**
    - **Labor Policies**
  - **Capacity Plan**
- **AGGREGATE PLANNING**
  - **Aggregate Plan**
- **WIP/QUOTA SETTING**
  - **Master Production Schedule**
- **SEQUENCING & SCHEDULING**
  - **Work Schedule**
  - **SHOP FLOOR CONTROL**
  - **WIP Position**
- **REAL-TIME SIMULATION**
  - **Work Forecast**
- **DEMAND MANAGEMENT**
  - **Customer Demands**
  - **WIP/QUOTA SETTING**
  - **SEQUENCING & SCHEDULING**
  - **REAL-TIME SIMULATION**

Strategy

Tactics

Control

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Basic Aggregate Planning

Problem: project production of single product over planning horizon.

Motivation for Study:

- mechanics and value of LP as a tool
- intuition of production smoothing

Inputs:

- demand forecast (over planning horizon)
- capacity constraints
- unit profit
- inventory carrying cost rate
A Simple AP Model

Notation:

\( t \) = an index of the time periods, \( t = 1, \ldots, \bar{t} \).

\( d_t \) = demand in period \( t \).

\( c_t \) = capacity in period \( t \).

\( r \) = unit profit (not including holding cost)

\( h \) = cost to hold one unit of inventory for one period.

\( X_t \) = quantity produced during period \( t \).

\( S_t \) = quantity sold during period \( t \).

\( I_t \) = inventory at the end of period \( t \).
A Simple AP Model (cont.)

Formulation

\[
\max \sum_{t=1}^{\bar{t}} rS_t - hI_t \quad \text{sales revenue - holding cost}
\]

subject to

\[
S_t \leq d_t \quad t = 1, \ldots, \bar{t} \quad \text{demand}
\]

\[
X_t \leq c_t \quad t = 1, \ldots, \bar{t} \quad \text{capacity}
\]

\[
I_t = I_{t-1} + X_t - S_t, \quad t = 1, \ldots, \bar{t} \quad \text{inventory balance}
\]

\[
X_t, S_t, I_t \geq 0 \quad t = 1, \ldots, \bar{t} \quad \text{non-negativity}
\]
A Simple AP Example

Data:  

\[
\begin{align*}
r &= $10 \\
h &= $1 \\
I_0 &= 0 \\
c_t &= 100, \ t = 1, \ldots, 6 \\
d_t &= 80, 100, 120, 140, 90, 140
\end{align*}
\]

Optimal Solution:

<table>
<thead>
<tr>
<th>( t )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_t )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>120</td>
<td>110</td>
<td>120</td>
</tr>
<tr>
<td>( S_t )</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>120</td>
<td>90</td>
<td>140</td>
</tr>
<tr>
<td>( I_t )</td>
<td>20</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>
A Simple AP Example (cont.)

Interpretation

- solution
- shadow prices
- allowable increases / decreases
Product Mix Planning

Problem: **determine most profitable mix over planning horizon**

Motivation for Study:
- linking marketing/promotion to logistics.
- Bottleneck identification.

Inputs:
- demand forecast by product (family?); may be ranges
- unit hour data
- capacity constraints
- unit profit by product
- holding cost
Basic Verbal Formulation

maximize \textit{profit}

subject to:

\textit{production} \leq \textit{capacity}, \quad \text{at all workstations in all periods}

\textit{sales} \leq \textit{demand}, \quad \text{for all products in all periods}

\textit{Note:} we will need some technical constraints to ensure that variables represent reality.
Product Mix Notation

\[ \begin{align*}
i &\quad = \text{an index of product, } i = 1,\ldots, m \\
j &\quad = \text{an index of workstation, } j = 1,\ldots, n \\
t &\quad = \text{an index of period, } t = 1,\ldots, \bar{t} \\
\bar{d}_{it} &\quad = \text{maximum demand for product } i \text{ in period } t. \\
d_{it} &\quad = \text{minimum sales allowed of product } i \text{ in period } t \\
a_{ij} &\quad = \text{time required on workstation } j \text{ to produce one unit of product } i. \\
c_{jt} &\quad = \text{capacity of workstation } j \text{ in period } t. \\
r_i &\quad = \text{net profit from one unit of product } i \\
h_i &\quad = \text{cost to hold one unit of } i \text{ for one period } t. \\
X_{it} &\quad = \text{amount of product } i \text{ produced in period } t \\
S_{it} &\quad = \text{amount of product } i \text{ sold in period } t. \\
I_{it} &\quad = \text{inventory of product } i \text{ at end of } t. \\
\end{align*} \]
Product Mix Formulation

\[
\max \sum_{t=1}^{\tilde{t}} \sum_{i=1}^{m} r_i S_{it} - h_i I_{it} \quad \text{sales revenue - holding cost}
\]

subject to
\[
\begin{align*}
    & d_{it} \leq S_{it} \leq \bar{d}_{it} \quad \text{for all } i,t \quad \text{demand} \\
    & \sum_{i=1}^{m} a_{ij} X_{it} \leq c_{jt} \quad \text{for all } j,t \quad \text{capacity} \\
    & I_{it} = I_{i,t-1} + X_{it} - S_{it}, \quad \text{for all } i,t \quad \text{inventory balance} \\
    & X_{it}, S_{it}, I_{it} \geq 0 \quad \text{for all } i,t \quad \text{non-negativity}
\end{align*}
\]
Product Mix (Goldratt) Example

Assumptions:

- two products, \( P \) and \( Q \)
- constant weekly demand, cost, capacity, etc.
- **Objective:** maximize weekly profit

Data:

<table>
<thead>
<tr>
<th>Product</th>
<th>( P )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>$90</td>
<td>$100</td>
</tr>
<tr>
<td>Raw Material Cost</td>
<td>$45</td>
<td>$40</td>
</tr>
<tr>
<td>Max Weekly Sales</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter A</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter B</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter C</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter D</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>
A Cost Approach

Unit Profit

Product $P$ : $45$
Product $Q$ : $60$

Maximum Production of $Q$ : **50 units**

Available Capacity for Producing $P$

\[
\begin{align*}
2400 - 10 (50) &= 1,900 \text{ minutes on Workcenter A} \\
2400 - 30 (50) &= 900 \text{ minutes on Workcenter B} \\
2400 - 5 (50) &= 2,150 \text{ minutes on Workcenter C} \\
2400 - 5 (50) &= 2,150 \text{ minutes on Workcenter D}
\end{align*}
\]

Maximum Production of $P$: **900/15 = 60 units**

Net Weekly Profit: $45 \times 60 + 60 \times 50 - 5,000 = 700$
A Bottleneck Approach

Identifying the Bottleneck: Workcenter B, because

- 15 (100) + 10 (50) = 2,000 minutes on workcenter A
- 15 (100) + 30 (50) = 3,000 minutes on workcenter B
- 15 (100) + 5 (50) = 1,750 minutes on workcenter C
- 15 (100) + 5 (50) = 1,750 minutes on workcenter D

Profit per Minute of Bottleneck Time used:

- $45/15 = $3 per minute spent processing P
- $60/30 = $2 per minute spent processing Q

Maximum Production of P: 100 units

Maximum Production of Q: 900/30 = 30 units

Net Weekly Profit: $45 \times 100 + $60 \times 30 - $5,000 = $1,300
A Modified Example

Changes: in processing times on workcenters B and D.

Data:

<table>
<thead>
<tr>
<th>Product</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>$90</td>
<td>$100</td>
</tr>
<tr>
<td>Raw Material Cost</td>
<td>$45</td>
<td>$40</td>
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<tr>
<td>Max Weekly Sales</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter A</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter B</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter C</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>Minutes per unit on Workcenter D</td>
<td>25</td>
<td>14</td>
</tr>
</tbody>
</table>
A Bottleneck Approach

Identifying the Bottleneck: Workcenter B, because

- $15 \times 100 + 10 \times 50 = 2,000$ minutes on workcenter A
- $15 \times 100 + 35 \times 50 = 3,250$ minutes on workcenter B
- $15 \times 100 + 5 \times 50 = 1,750$ minutes on workcenter C
- $25 \times 100 + 14 \times 50 = 3,200$ minutes on workcenter D

Bottleneck at B:

- $\frac{45}{15} = $3 per minute spent processing $P$
- $\frac{60}{35} = $1.71 per minute spent processing $Q$

Maximum Production of $P$: $\frac{2400}{25} = 96$ units

Maximum Production of $Q$: 0 units

Net Weekly Profit: $45 \times 96 - 5,000 = -$680
A Bottleneck Approach (cont.)

Bottleneck at D:

$\frac{45}{25} = 1.80$ per minute spent processing $P$

$\frac{60}{14} = 4.29$ per minute spent processing $Q$

Maximum Production of $Q$: $\frac{2400}{35} = 68.57 > 50$, produce 50

Available time on Bottleneck:

$2400 - 14(50) = 1,700$ minutes on workcenter D

Maximum Production of $P$: $\frac{1700}{25} = 68$ units

Net Weekly Profit: $45 \times 43 + 60 \times 50 - 5000 = -65$
An LP Approach

Formulation: \[ \text{max } 45X_p + 60X_Q - 5000 \]
subject to:
\[ 15X_p + 10X_Q \leq 2400 \]
\[ 15X_p + 35X_Q \leq 2400 \]
\[ 15X_p + 5X_Q \leq 2400 \]
\[ 25X_p + 14X_Q \leq 2400 \]

Solution: Optimal Objective \( = \$557.94 \)
\[ X_p^* = 75.79 \]
\[ X_Q^* = 36.09 \]

Net Weekly Profit: Round solution down (still feasible) to:
\[ X_p^* = 75 \]
\[ X_Q^* = 36 \]
To get \( \$45 \times 75 + \$60 \times 36 - \$5,000 = \$535. \)
Extensions to Basic Product Mix Model

Other Resource Constraints:

Notation:

\[ b_{ij} = \text{units of resource } j \text{ required per unit of product } i \]
\[ k_{jt} = \text{number of units of resource } j \text{ available in period } t \]
\[ X_{it} = \text{amount of product } i \text{ produced in period } t \]

Constraint for Resource \( j \):

\[ \sum_{i=1}^{m} b_{ij} X_{it} \leq k_{jt} \]

Utilization Matching: Let \( q \) represent fraction of rated capacity we are willing to run on resource \( j \).

\[ \sum_{i=1}^{m} a_{ij} X_{it} \leq q c_{jt} \text{ for all } j, t \]
Extensions to Basic Product Mix Model (cont.)

Backorders:

- Substitute \( I_{it} = I_{it}^+ - I_{it}^- \)
- Allow \( I_{it} \) to become positive or negative
- Penalize \( I_{it}^+ , I_{it}^- \) differently in objective if desired

Overtime:

- Define \( O_{jt} \) as hours of OT used on resource \( j \) in period \( t \)
- Add \( O_{jt} \) to \( c_{jt} \) in capacity constraint.
- Penalize \( O_{jt} \) in objective if desired
Workforce Planning

Problem: determine most profitable production and hiring/firing policy over planning horizon.

Motivation for Study:
- hiring/firing vs. overtime vs. Inventory Build tradeoff
- iterative nature of optimization modeling.

Inputs:
- demand forecast (assume single product for simplicity)
- unit hour data
- labor content data
- capacity constraints
- hiring/firing costs
- overtime costs
- holding costs
- unit profit
Workforce Planning Notation

\[ j = \text{an index of workstation, } j = 1, \ldots, n \]
\[ t = \text{an index of period, } t = 1, \ldots, \bar{t} \]
\[ \bar{d}_t = \text{maximum demand in period } t. \]
\[ \underline{d}_t = \text{minimum sales allowed in period } t \]
\[ a_j = \text{unit hours on workstation } j \]
\[ b = \text{number of man hours required to produce one unit.} \]
\[ c_{jt} = \text{capacity of work center } j \text{ in period } t. \]
\[ r = \text{net profit from one unit.} \]
\[ h = \text{cost to hold one unit for one period } t. \]
\[ l = \text{cost of regular time in dollars/man-hour} \]
\[ l' = \text{cost of overtime in dollars/man-hour} \]
\[ e = \text{cost to increase workforce by one man-hour} \]
\[ e' = \text{cost to decrease workforce by one man-hour} \]
Workforce Planning Notation (cont.)

\[ X_t = \text{amount produced in period } t \]
\[ S_t = \text{amount sold in period } t \]
\[ I_t = \text{inventory at end of } t \]
\[ W_t = \text{workforce period } t \text{ in man-hours of regular time} \]
\[ H_t = \text{increase (hires) in workforce from period } t - 1 \text{ to } t \text{ in man-hours.} \]
\[ F_t = \text{decrease (fires) in workforce from period } t - 1 \text{ to } t \text{ in man-hours.} \]
\[ O_t = \text{overtime in period } t \text{ in hours} \]
Workforce Planning Formulation

\[
\max \sum_{t=1}^{T} \left\{ rS_t - hI_t - lW_t - l'O_t - eH_t - e'F_t \right\}
\]

subject to
\[
\begin{align*}
    d_l & \leq S_t \leq d_u & \text{for all } t \\
    a_j X_t & \leq c_{jt} & \text{for all } t \\
    I_t &= I_{t-1} + X_t - S_t, & \text{for all } t \\
    W_t &= W_{t-1} + H_t - F_t & \text{for all } t \\
    bX_t & \leq W_t + O_t & \text{for all } t \\
    X_t, S_t, I_t, O_t, W_t, H_t, F_t & \geq 0 & \text{for all } t
\end{align*}
\]
Workforce Planning Example

**Problem Description**

- 12 month planning horizon
- 168 hours per month
- 15 workers currently in system
- regular time labor at $35 per hour
- overtime labor at $52.50 per hour
- $2,500 to hire and train new worker
  \[ \frac{2,500}{168} = \frac{14.88}{\text{hour}} \approx \$15/\text{hour} \]
- $1,500 to lay off worker
  \[ \frac{1,500}{168} = \frac{8.93}{\text{hour}} \approx \$9/\text{hour} \]
- 12 hours labor per unit
- demand assumed met \( S_t = d_t \) so \( S_t \) variables are unnecessary
Solutions:

• “Chase” Solution: infeasible
• LP optimal Solution: layoff 9.5 workers
• Add constraint: $F_t=0$
  – results in 48 hours/worker/week of overtime
• Add constraint: $O_t \leq 0.2W_t$
  – Reasonable solution?
Conclusions

No single AP model is right for every situation

Simplicity promotes understanding

Linear programming is a useful AP tool

Robustness matters more than precision

Formulation and Solution are not separate activities.
Inventory Management

_One's work may be finished some day, but one's education never._

– Alexandre Dumas
Hierarchical Planning – Roles of Inventory

Marketing Parameters → FORECASTING → aggregation, postponement, etc.
Product/Process Parameters → CAPACITY/FACILITY PLANNING
Capacity Plan → WORKFORCE PLANNING → WORKFORCE Plan → Labor Policies

Strategy
- flexibility, teaming
- seasonal build, inv/OT tradeoff

buffer sizes
- build-ahead, batching, safety stock, etc.

Tactics
- flow control - push/pull, etc.
- WIP tracking

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Inventory is the Lifeblood of Manufacturing

Plays a role in almost all operations decisions
  • shop floor control
  • scheduling
  • aggregate planning
  • capacity planning, …

Links to most other major strategic decisions
  • quality assurance
  • product design
  • facility design
  • marketing
  • organizational management, …

Managing inventory is close to managing the entire system…
Plan of Attack

Classification:
- raw materials
- work-in-process (WIP)
- finished goods inventory (FGI)
- spare parts

Justification:
- Why is inventory being held?
- benchmarking
Plan of Attack (cont.)

Structural Changes:

• major reorganization (e.g., eliminate stockpoints, change purchasing contracts, alter product mix, focused factories, etc.)
• reconsider objectives (e.g., make-to-stock vs. make-to-order, capacity strategy, time-based-competition, etc.)

Modeling:

• What to model – identify key tradeoffs.
• How to model – EOQ, (Q,r), optimization, simulation, etc.
Raw Materials

**Reasons for Inventory:**
- batching (quantity discounts, purchasing capacity, …)
- safety stock (buffer against randomness in supply/production)
- obsolescence

**Improvement Policies:**
- Pareto analysis (focus on 20% of parts that represent 80% of $ value)
- ABC classification (stratify parts management)
- JIT deliveries (expensive and/or bulky items)
- vendor monitoring/management
Raw Materials (cont.)

Benchmarks:
- small C parts: 4-8 turns
- A,B parts: 12-25+ turns
- bulky parts: up to 50+ turns

Models:
- EOQ
- power-of-two
- service constrained optimization model
Multiproduct EOQ Models

**Notation:**

- \( N \) = total number of distinct part numbers in the system
- \( D_i \) = demand rate (units per year) for part \( i \)
- \( c_i \) = unit production cost of part \( i \)
- \( A_i \) = fixed cost to place an order for part \( i \)
- \( h_i \) = cost to hold one unit of part \( i \) for one year
- \( Q_i \) = the size of the order or lot size for part \( i \) (decision variable)
Multiproduct EOQ Models (cont.)

**Cost-Based EOQ Model:** For part $i$,

$$Q_i^* = \sqrt{\frac{2AD_i}{h_i}}$$

but what is $A$?

**Frequency Constrained EOQ Model:**

$$\min \quad \text{Inventory holding cost}$$
subject to:

Average order frequency $\leq F$
Multiproduct EOQ Solution Approach

Constraint Formulation:

\[
\min \sum_{i=1}^{N} \frac{h_i Q_i}{2}
\]

subject to:

\[
\frac{1}{N} \sum_{i=1}^{N} \frac{D_i}{Q_i} \leq F
\]

Cost Formulation:

\[
Y(Q) = \sum_{i=1}^{N} \frac{h_i Q_i}{2} + \sum_{i=1}^{N} \frac{AD_i}{Q_i}
\]
Multiproduct EOQ Solution Approach (cont.)

Cost Solution: Differentiate $Y(Q)$ with respect to $Q_i$, set equal to zero, and solve:

$$\frac{\partial Y}{\partial Q_i} = \frac{h_i}{2} - \frac{AD_i}{Q_i^2}$$

$$Q_i(A) = \sqrt{\frac{2AD_i}{h_i}} \quad \text{No surprise - regular EOQ formula}$$

Constraint Solution: For a given $A$ we can find $Q_i(A)$ using the above formula. The resulting average order frequency is:

$$F(A) = \frac{1}{N} \sum_{i=1}^{N} \frac{D_i}{Q_i(A)}$$

If $F(A) < F$ then penalty on order frequency is too high and should be decreased. If $F(A) > F$ then penalty is too low and needs to be increased.
Multiproduct EOQ Procedure – Constrained Case

Step (0) Establish a tolerance for satisfying the constraint (i.e., a sufficiently small number that represents “close enough” for the order frequency) and guess a value for \( A \).

Step (1) Use \( A \) in previous formula to compute \( Q_i(A) \) for \( i = 1, \ldots, N \).

Step (2) Compute the resulting order frequency:

\[
F(A) = \frac{1}{N} \sum_{i=1}^{N} \frac{D_i}{Q_i(A)}
\]

If \( |F(A) - F| < \varepsilon \), STOP; \( Q_i^* = Q_i(A) \), \( i = 1, \ldots, N \). ELSE,

If \( F(A) < F \), decrease \( A \)
If \( F(A) < F \), increase \( A \)
Go to Step (1).

Note: The increases and decreases in \( A \) can be made by trial and error, or some more sophisticated search technique, such as interval bisection.
Multiproduct EOQ Example

Input Data:

<table>
<thead>
<tr>
<th>Part</th>
<th>$D_i$</th>
<th>$c_i$</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>1000</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
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</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>10</td>
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**Multiproduct EOQ Example (cont.)**

Calculations:

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<th>A</th>
<th>$Q_1(A)$</th>
<th>$Q_2(A)$</th>
<th>$Q_3(A)$</th>
<th>$Q_4(A)$</th>
<th>$F(A)$</th>
<th>Inventory Investment</th>
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<td>114.14</td>
<td>11.41</td>
<td>36.09</td>
<td>12.00</td>
<td>3,126.53</td>
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</table>
Powers-of-Two Adjustment

Rounding Order Intervals:

\[
T_1^* = \frac{Q_1^*}{D_1} = \frac{36.09}{1000} = 0.03609 \text{ yrs} = 13.17 \approx 16 \text{ days}
\]

\[
T_2^* = \frac{Q_2^*}{D_2} = \frac{114.14}{1000} = 0.11414 \text{ yrs} = 41.66 \approx 32 \text{ days}
\]

\[
T_3^* = \frac{Q_3^*}{D_3} = \frac{11.41}{100} = 0.11414 \text{ yrs} = 41.66 \approx 32 \text{ days}
\]

\[
T_4^* = \frac{Q_4^*}{D_4} = \frac{36.09}{100} = 0.3609 \text{ yrs} = 131.73 \approx 128 \text{ days}
\]

Rounded Order Quantities:

\[
Q_1' = D_1 \frac{T_1'}{365} = 1000 \times \frac{16}{365} = 43.84
\]

\[
Q_2' = D_2 \frac{T_2'}{365} = 1000 \times \frac{32}{365} = 87.67
\]

\[
Q_3' = D_3 \frac{T_3'}{365} = 100 \times \frac{32}{365} = 8.77
\]

\[
Q_4' = D_4 \frac{T_4'}{365} = 100 \times \frac{128}{365} = 35.07
\]
Powers-of-Two Adjustment (cont.)

Resulting Inventory and Order Frequency: Optimal inventory investment is $3,126.53 and order frequency is 12. After rounding to nearest powers-of-two, we get:

\[
\text{Inventory Investment} = \sum_{i=1}^{4} \frac{c_i Q'_i}{2} = 3,243.84
\]

\[
\text{Average Order Frequency} = \frac{1}{4} \sum_{i=1}^{4} \frac{D_i}{Q'_i} = 12.12
\]
Questions – Raw Materials

- Do you track vendor performance (i.e., as to variability)?
- Do you have a vendor certification program?
- Do your vendor contracts have provisions for varying quantities?
- Are purchasing procedures different for different part categories?
- Do you make use of JIT deliveries?
- Do you have excessive “wait to match” inventory? (May need more safety stock of inexpensive parts.)
- Do you have too many vendors?
- Is current order frequency rationalized?
Work-in-Process

Reasons for Inventory:

• queueing (variability)
• processing
• waiting to move (batching)
• moving
• waiting to match (synchronization)
Work-in-Process (cont.)

Improvement Policies:

- pull systems
- synchronization schemes
- lot splitting
- flow-oriented layout, floating work
- setup reduction
- reliability/maintainability upgrades
- focused factories
- improved yield/rework
- better scheduling
- judicious vending
Work-in-Process (cont.)

Benchmarks:
- coefficients of variation below one
- WIP below 10 times critical WIP
- relative benchmarks depend on position in supply chain

Models:
- queueing models
- simulation
Science Behind WIP Reduction

Cycle Time:

\[ CT \approx \left[ \left( \frac{c_a^2 + c_e^2}{2} \right) \left( \frac{u}{1-u} \right) t_e \right] + t_e \]

WIP:

\[ WIP = CT \cdot TH \approx \left[ \left( \frac{c_a^2 + c_e^2}{2} \right) \left( \frac{u}{1-u} \right) u \right] + u \]

Conclusion: CT and WIP can be reduced by reducing utilization, variability, or both.
Questions – WIP

• Are you using production leveling and due date negotiation to smooth releases?
• Do you have long, infrequent outages on machines?
• Do you have long setup times on highly utilized machines?
• Do you move product infrequently in large batches?
• Do some machines have utilizations in excess of 95%?
• Do you have significant yield/rework problems?
• Do you have significant waiting inventory at assembly stations (i.e., synchronization problems)?
Finished Goods Inventory

Reasons for Inventory:

• respond to variable customer demand
• absorb variability in cycle times
• build for seasonality
• forecast errors

Improvement Policies:

• dynamic lead time quoting
• cycle time reduction
• cycle time variability reduction
• late customization
• balancing labor/inventory
• improved forecasting
Finished Goods Inventory (cont.)

Benchmarks:

- seasonal products: 6-12 turns
- make-to-order products: 30-50+ turns
- make-to-stock products: 12-24 turns

Models:

- reorder point models
- queueing models
- simulation
Questions – FGI

• All the WIP questions apply here as well.

• Are lead times negotiated dynamically?

• Have you exploited opportunities for late customization (e.g., bank stocks, product standardization, etc.)?

• Have you adequately considered variable labor (seasonal hiring, cross-trained workers, overtime)?

• Have you evaluated your forecasting procedures against past performance?
Spare Parts Inventory

Reasons for Inventory:
- customer service
- purchasing/production lead times
- batch replenishment

Improvement Policies:
- separate scheduled/unscheduled demand
- increase order frequency
- eliminate unnecessary safety stock
- differentiate parts with respect to fill rate/order frequency
- forecast life cycle effects on demand
- balance hierarchical inventories
Spare Parts Inventory (cont.)

Benchmarks:
- scheduled demand parts: 6-24 turns
- unscheduled demand parts: 1-12+ turns (highly variable!)
- Wharton survey

Models:
- \((Q,r)\)
- distribution requirements planning (DRP)
- multi-echelon models
Multi-Product \((Q,r)\) Systems

Many inventory systems (including most spare parts systems) involve multiple products (parts)

Products are not always separable because:
- average service is a function of all products
- cost of holding inventory is different for different products

Different formulations are possible, including:
- constraint formulation (usually more intuitive)
- cost formulation (easier to model, can be equivalent to constraint approach)
Model Inputs and Outputs

**Costs**
- Order (A)
- Backorder (b) or Stockout (k)
- Holding (h)

**Inputs (by part)**
- Cost (c)
- Mean LT demand (θ)
- Std Dev of LT demand (σ)

**Stocking Parameters (by part)**
- Order Quantity (Q)
- Reorder Point (r)

**Performance Measures (by part and for system)**
- Order Frequency (F)
- Fill Rate (S)
- Backorder Level (B)
- Inventory Level (I)
Multi-Prod \((Q,r)\) Systems – Constraint Formulations

**Backorder model**

\[
\begin{align*}
\text{min} & \quad \text{Inventory investment} \\
\text{subject to:} & \quad \text{Average order frequency} \leq F \\
& \quad \text{Average backorder level} \leq B
\end{align*}
\]

**Fill rate model**

\[
\begin{align*}
\text{min} & \quad \text{Inventory investment} \\
\text{subject to:} & \quad \text{Average order frequency} \leq F \\
& \quad \text{Average fill rate} \geq S
\end{align*}
\]

Two different ways to represent customer service.
Multi Product \((Q, r)\) Notation

\[ N = \text{number of distinct parts in the system} \]
\[ D_i = \text{expected demand per year for part } i \]
\[ D_{\text{tot}} = \sum_{i=1}^{N} D_i = \text{total demand} \]
\[ \ell_i = \text{replenishment lead time (assumed constant) for part } i \]
\[ \theta_i = \text{expected demand during replenishment lead time for part } i \]
\[ \sigma_i = \text{standard deviation of demand during replenishment lead time for part } i \]
\[ p_i(x) = \text{pmf of demand during lead time for part } i \]
\[ G_i(x) = \text{cdf of demand during lead time for part } i \]
\[ c_i = \text{unit cost of for part } i \]
\[ h_i = \text{annual unit holding cost for part } i \]
\[ A = \text{fixed cost per order} \]
\[ b = \text{annual unit backorder cost (all parts)} \]
\[ k = \text{cost per stockout (all parts)} \]
Multi-Product \((Q,r)\) Notation (cont.)

Decision Variables:

\[ Q_i = \text{order quantity for part } i \]
\[ r_i = \text{reorder point for part } i \]
\[ s_i = r_i - \theta_i = \text{safety stock implied by } r_i \]

Performance Measures:

\[ F_i(Q_i, r_i) = \text{average order frequency for part } i \]
\[ S_i(Q_i, r_i) = \text{average service level (fill rate) for part } i \]
\[ B_i(Q_i, r_i) = \text{average backorder level for part } i \]
\[ I_i(Q_i, r_i) = \text{average inventory level for part } i \]
Backorder Constraint Formulation

Verbal Formulation:

\[ \text{min} \quad \text{Inventory investment} \]
subject to: \[ \text{Average order frequency} \leq F \]
\[ \text{Total backorder level} \leq B \]

Mathematical Formulation:

\[ \text{min} \quad \sum_{i=1}^{N} c_i I_i(Q_i, r_i) \]
subject to: \[ \frac{1}{N} \sum_{i=1}^{N} F_i(Q_i, r_i) \leq F \]
\[ \sum_{i=1}^{N} B_i(Q_i, r_i) \leq B \]

"Coupling" of \( Q \) and \( r \) makes this hard to solve.
Backorder Cost Formulation

Verbal Formulation:

\[ \text{min} \ \text{Ordering Cost} + \text{Backorder Cost} + \text{Holding Cost} \]

Mathematical Formulation:

\[ \text{min} \ \sum_{i=1}^{N} \{ AF_i (Q_i, r_i) + BB_i (Q_i, r_i) + h_i I_i (Q_i, r_i) \} \]

“Coupling” of \( Q \) and \( r \) makes this hard to solve.
Fill Rate Constraint Formulation

Verbal Formulation:

\[ \text{min} \quad \text{Inventory investment} \]
\[ \text{subject to:} \quad \text{Average order frequency} \leq F \]
\[ \text{Average fill rate} \geq S \]

Mathematical Formulation:

\[ \text{min} \quad \sum_{i=1}^{N} c_i I_i(Q_i, r_i) \]
\[ \text{subject to:} \quad \frac{1}{N} \sum_{i=1}^{N} F_i(Q_i, r_i) \leq F \]
\[ \frac{1}{D_{\text{tot}}} \sum_{i=1}^{N} D_i S_i(Q_i, r_i) \geq S \]

“Coupling” of \( Q \) and \( r \) makes this hard to solve.
Fill Rate Cost Formulation

**Verbal Formulation:**

\[
\min \text{ Ordering Cost + Stockout Cost + Holding Cost}
\]

**Mathematical Formulation:**

\[
\min \sum_{i=1}^{N} \{ AF_i(Q_i, r_i) + kD_i(1 - S_i(Q_i, r_i)) + h_iI_i(Q_i, r_i) \}
\]

*Note:* a stockout cost penalizes each order not filled from stock by \( k \) regardless of the duration of the stockout.

*“Coupling” of \( Q \) and \( r \) makes this hard to solve.*
Relationship Between Cost and Constraint Formulations

Method:
1) Use cost model to find $Q_i$ and $r_i$, but keep track of average order frequency and fill rate using formulas from constraint model.

2) Vary order cost $A$ until order frequency constraint is satisfied, then vary backorder cost $b$ (stockout cost $k$) until backorder (fill rate) constraint is satisfied.

Problems:
• Even with cost model, these are often a large-scale integer nonlinear optimization problems, which are hard.
• Because $B_i(Q_i, r_i)$, $S_i(Q_i, r_i)$, $I_i(Q_i, r_i)$ depend on both $Q_i$ and $r_i$, solution will be “coupled”, so step (2) above won’t work without iteration between $A$ and $b$ (or $k$).
Type I (Base Stock) Approximation for Backorder Model

Approximation:

- replace $B_i(Q_i, r_i)$ with base stock formula for average backorder level, $B(r_i)$
- Note that this “decouples” $Q_i$ from $r_i$ because $F_i(Q_i, r_i) = D_i/Q_i$ depends only on $Q_i$ and not $r_i$

Resulting Model:

$$\min \sum_{i=1}^{N} \left\{ A \frac{D_i}{Q_i} + bB(r_i) + h_i \left( \frac{Q_i + 1}{2} + r_i - \theta_i + B(r_i) \right) \right\}$$
Solution of Approximate Backorder Model

Taking derivative with respect to $Q_i$ and solving yields:

$$Q_i^* = \sqrt{\frac{2AD_i}{h_i}} \quad \text{EOQ formula again}$$

Taking derivative with respect to $r_i$ and solving yields:

$$G(r_i^*) = \frac{b}{h_i + b} \quad \Rightarrow \quad r_i^* = \theta_i + z_i\sigma_i \quad \text{base stock formula again}$$

if $G_i$ is normal($\theta_i, \sigma_i$),
where $\Phi(z_i) = b/(h_i + b)$
Using Approximate Cost Solution to Get a Solution to the Constraint Formulation

1) Pick initial $A$, $b$ values.
2) Solve for $Q_i$, $r_i$ using:

$$Q_i^* = \sqrt{\frac{2AD_i}{h_i}}$$

$$r_i^* = \theta_i + z_i\sigma_i$$

3) Compute average order frequency and backorder level:

$$\bar{F} = \frac{1}{N} \sum_{i=1}^{N} \frac{D_i}{Q_i}$$

$$B_{tot} = \sum_{i=1}^{N} B_i(Q_i, r_i)$$  \textit{Note: use exact formula for $B(Q_i, r_i)$ not approx.}

4) Adjust $A$ until
Adjust $b$ until

$$\bar{F} = F$$

$$B_{tot} = B$$  \textit{Note: search can be automated with Solver in Excel.}
Type I and II Approximation for Fill Rate Model

Approximation:

• Use EOQ to compute $Q_i$ as before
• Replace $B_i(Q_i,r_i)$ with $B(r_i)$ (Type I approx) in inventory cost term.
• Replace $S_i(Q_i,r_i)$ with $1-B(r_i)/Q_i$ (Type II approx) in stockout term

Resulting Model:

$$\min \sum_{i=1}^{N} \left\{ A \frac{D_i}{Q_i} + kD \frac{B(r_i)}{Q_i} + h_i \left( \frac{Q_i + 1}{2} + r_i - \theta_i + B(r_i) \right) \right\}$$

Note: we use this approximate cost function to compute $r_i$ only, not $Q_i$
Solution of Approximate Fill Rate Model

EOQ formula for $Q_i$ yields:

$$Q_i^* = \sqrt{\frac{2AD_i}{h_i}}$$

Taking derivative with respect to $r_i$ and solving yields:

$$G(r_i^*) = \frac{kD_i}{kD_i + hQ_i} \Rightarrow r_i^* = \theta_i + z_i \sigma_i$$

Note: modified version of basestock formula, which involves $Q_i$

if $G_i$ is normal($\theta_i, \sigma_i$),
where $\Phi(z_i) = kD_i/(kD_i + hQ_i)$
Using Approximate Cost Solution to Get a Solution to the Constraint Formulation

1) Pick initial $A$, $k$ values.
2) Solve for $Q_i$, $r_i$ using:
   
   $Q_i^* = \sqrt{\frac{2AD_i}{h_i}}$

   $r_i^* = \theta_i + z_i\sigma_i$

3) Compute average order frequency and fill rate using:
   
   $\bar{F} = \frac{1}{N} \sum_{i=1}^{N} \frac{D_i}{Q_i}$

   $\bar{S} = \frac{1}{D_{\text{tot}}} \sum_{i=1}^{N} D_i S(Q_i, r_i)$

   Note: use exact formula for $S(Q_i, r_i)$ not approx.

4) Adjust $A$ until $\bar{F} = F$
   Adjust $b$ until $\bar{S} = S$

   Note: search can be automated with Solver in Excel.
Multi-Product \((Q,r)\) Insights

- All other things being equal, an optimal solution will hold less inventory (i.e., smaller \(Q\) and \(r\)) for an expensive part than for an expensive one.

- Reduction in total inventory investment resulting from use of “optimized” solution instead of constant service (i.e., same fill rate for all parts) can be substantial.

- Aggregate service may not always be valid:
  - could lead to undesirable impacts on some customers
  - additional constraints (minimum stock or service) may be appropriate
Questions – Spare Parts Inventory

• Is scheduled demand handled separately from unscheduled demand?
• Are stocking rules sensitive to demand, replenishment lead time, and cost?
• Can you predict life-cycle demand better? Are you relying on historical usage only?
• Are your replenishment lead times accurate?
• Is excess distributed inventory returned from regional facilities to central warehouse?
• How are regional facility managers evaluated against inventory? Frequency of inspection?
• Are lateral transhipments between regional facilities being used effectively? Officially?
Multi-Echelon Inventory Systems

Questions:

• How much to stock?
• Where to stock it?
• How to coordinate levels?
Types of Multi-Echelon Systems

Level 1
- Serial System

Level 2
- General Arborescent System

Level 3
- Stocking Site

Inventory Flow
Two Echelon System

Warehouse
- evaluate with (Q,r) model
- compute stocking parameters and performance measures

Facilities
- evaluate with base stock model (ensures one-at-a-time demands at warehouse)
- consider delays due to stockouts at warehouse in replenishment lead times
Facility Notation

\[ D_{im} = \text{expected demand per year for part } i \text{ at facility } m \]
\[ \theta_{im} = \text{expected demand for part } i \text{ during replenishment lead time at facility } m \]
\[ \sigma_{im} = \text{standard deviation of demand for part } i \text{ during replenishment leadtime at facility } m \]
\[ d_{im} = \text{mean daily demand for part } i \text{ at facility } m \]
\[ \sigma_{im}(D) = \text{standard deviation of daily demand for part } i \text{ at facility } m \]
\[ p_{im}(x) = \text{pmf of demand during lead time for part } i \text{ at facility } m \]
\[ G_{im}(x) = \text{cdf of demand during lead time for part } i \text{ at facility } m \]
\[ W_i = \text{expected time an order for part } i \text{ waits at warehouse due to backordering} \]
\[ L_{im} = \text{lead time (including backorder delay) for an order of part } i \text{ from facility } m, \text{ a random variable} \]
Warehouse Notation

\( N \) = total number of distinct parts in the system
\( M \) = number of facilities serviced by warehouse

\[ D_i = \sum_{m=1}^{M} D_{im} \] = annual demand for part \( i \) at the warehouse

\( \ell_i \) = replenishment lead time for part \( i \) to the warehouse
\( \theta_i \) = expected demand for part \( i \) during replenishment lead time at warehouse
\( \sigma_i \) = std dev of demand for part \( i \) during replenishment lead time at warehouse

\( p_i(x) \) = pmf of demand during lead time for part \( i \) at warehouse
\( G_i(x) \) = cdf of demand during lead time for part \( i \) at warehouse

\( c_i \) = unit cost of for part \( i \)
\( h_i \) = unit holding cost for part \( i \)
\( b_i \) = unit backorder cost for part \( i \)

\( A \) = fixed setup cost
Variables and Measures in Two Echelon Model

**Decision Variables:**

\[ Q_i = \text{order quantity for part } i \text{ at warehouse} \]

\[ r_i = \text{reorder point for part } i \text{ at warehouse} \]

\[ r_{im} = \text{reorder point for part } i \text{ at facility } m \]

\[ R_{im} = \text{base stock level for part } i \text{ at facility } m \]

**Performance Measures:**

\[ F_i(Q_i, r_i) = \text{average order frequency for part } i \text{ at warehouse} \]

\[ S_i(Q_i, r_i) = \text{average service level (fill rate) for part } i \text{ at warehouse} \]

\[ B_i(Q_i, r_i) = \text{average backorder level for part } i \text{ at warehouse} \]

\[ I_i(Q_i, r_i) = \text{average inventory level for part } i \text{ at warehouse} \]

\[ S_{im}(R_{im}) = \text{average service level (fill rate) for part } i \text{ at facility } m \]

\[ B_{im}(R_{im}) = \text{average backorder level (fill rate) for part } i \text{ at facility } m \]

\[ I_{im}(R_{im}) = \text{average inventory level (fill rate) for part } i \text{ at facility } m \]
Facility Lead Times (mean)

Delay due to backordering:

\[ W_i = \frac{B_i(Q_i, r_i)}{D_i} \quad \text{by Little’s law} \]

Effective lead time for part \( i \) to facility \( m \):

\[ E[L_{im}] = \ell_{im} + W_i \]

\[ \theta_{im} = D_{im} E[L_{im}] \quad \text{use this in place of } \theta \text{ in base stock model for facilities} \]
Facility Lead Times (std dev)

If \( y = \) delay for an order that encounters stockout, then:

\[
E[L_{im}] = S_i (\ell_{im}) + (1 - S_i)(\ell_{im} + y)
\]

\[
y = \frac{W_i}{(1 - S_i)}
\]

**Variance of \( L_{im} \):**

\[
E[L_{im}^2] = S_i \ell_{im}^2 + (1 - S_i)(\ell_{im} + y)^2
\]

\[
\text{Var}(L_{im}) = E[L_{im}^2] - E[L_{im}]^2 = S_i (1 - S_i)y^2 = \frac{S_i}{1 - S_i}W_i^2
\]

\[
\sigma(L_{im}) = \sqrt{\frac{S_i}{1 - S_i}W_i}
\]

\[
\sigma_{im} = \sqrt{E[L_{im}]\sigma_{im}^2(D) + d_m^2\sigma^2(L_m)}
\]

Note: \( S_i = S_i(Q_b, r_i) \)

This just picks \( y \) to match mean, which we already know.

We can use this in place of \( \sigma \) in normal base stock model for facilities.
Two Echelon (Single Product) Example

*D* = 14 units per year (Poisson demand) at warehouse

*l* = 45 days

*Q* = 5

*r* = 3

*DM* = 7 units per year at a facility

*l*m* = 1 day (warehouse to facility)

\[
B(Q,r) = 0.0114
\]

\[
S(Q,r) = 0.9721
\]

\[
W = \frac{365B(Q,r)}{D} = 365(0.0114/14) = 0.296 \text{ days}
\]

\[
E[L_m] = 1 + 0.296 = 1.296 \text{ days}
\]

\[
\theta_m = D_m E[L_m] = (7/365)(1.296) = 0.0249 \text{ units}
\]

*single facility that accounts for half of annual demand*

*from previous example*
Two Echelon Example (cont.)

Standard deviation of demand during replenishment lead time:

\[
\sigma(L_m) = \sqrt{\frac{S}{1-S}} \approx 1.7474
\]

\[
W = \sqrt{\frac{0.9721}{1-0.9721}} \approx 0.296
\]

\[
\sigma_m = \sqrt{E[L]\sigma^2(D) + d_m^2\sigma^2(L_m)} = \sqrt{1.296(\sqrt{0.0192}) + (0.0192^2)(1.7474^2)} \approx 0.1612
\]

Backorder level:

\[
B(1) = 0.9225
\]

\[
B(2) \approx 1
\]

Computed from basestock model using \( \theta_m \) and \( \sigma_m \)

Conclusion: base stock level of 2 probably reasonable for facility.
Observations on Multi-Echelon Systems

• Service at central DC is a means to an ends (i.e., service at facilities).

• Service matters at locations that interface with customers:
  – fill rate (fraction of demands filled from stock)
  – average delay (expected wait for a part)

• Multi-echelon systems are hard to model/solve exactly, so we try to “decouple” levels.
  – Example: set fill rate at at DC and compute expected delay at facilities, then search over DC service to minimize system cost.

• Structural changes are an option
  – (e.g., change number of DC's or facilities, allow cross-sharing, have suppliers deliver directly to outlets, etc.)