1. Prove that the equation of motion

\[ M \frac{d^2 u_s}{dt^2} = C \left( u_{s+1} + u_{s-1} - 2u_s \right) \]

in the long wavelength limit \( Ka << 1 \) is equivalent to the continuum elastic wave equation

\[ \frac{\partial^2 u_s}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} \]

in which \( v \) is the velocity of sound.

2. Consider the normal modes of a linear chain in which the force constants between nearest-neighbor atoms are alternatively \( C \) and \( 10C \). Assuming that the masses are equal and the nearest neighbor separation is \( a/2 \) find (a) \( \omega(k) \); (b) the values of \( \omega(k) \) at \( k = 0 \) and \( k = \pi/a \); (c) long-wavelength limit of the spectrum.

3. Consider a linear chain with a basis of two different atoms with masses \( M_1 \) and \( M_2 \) in the approximation when only nearest neighbors interact and \( M_1 >> M_2 \).

(a) Find the ratio of amplitudes \( u/v \) for two branches at the Brillouin zone boundary \( K = K_{max} \).

(b) Discuss the form of the dispersion relation and the nature of the vibration modes.

4. Problem # 4, (Kittel, Chapter 4)