Math 120: Exam 4 Review

Sections 1.1 - 2.2, 2.4

Text: Trigonometry, Second Edition: Coburn and Herdlick

Pre-Test

Solutions to the following questions are given as Examples in the review sections that follow.

1. Convert $44.0805434^\circ$ to degrees/minutes/seconds (DMS).
2. Convert $44^\circ 4' 49.956''$ to decimal degrees.
3. Convert $-103^\circ 13' 51.6''$ to decimal degrees.
4. Find the coterminal angle whose measure is between $-180^\circ$ and $+180^\circ$ for each of the following.
   a) $\theta = 495^\circ$
   b) $\theta = 645^\circ$
5. For a 30-60-90 triangle, if the hypotenuse ($h$) is of length 10, find the lengths of the other sides.
6. For a 30-60-90 triangle, if the longer leg ($b$) measures $4\sqrt{3}$, find the lengths of the other sides.
7. Given the following angles of a right triangle, $\angle A = 3x$, $\angle B = (5x - 3)$, $\angle C = 47$, find the value of $x$.
8. Using Figure 1 find expressions for all sides of all triangles shown (four triangles in total).

![Figure 1: Similar triangles](image)

9. Given $\sin \theta = -\frac{3}{5}$, $\cot \theta < 0$, find the values of $x$, $y$ and $r$. Clearly indicate the quadrant of the terminal side of $\theta$, then state the values of the six trig functions of $\theta$.
10. Find the value of $x$ that makes $\cos(2x) = \sin(3x)$.
11. If $\tan(75^\circ) = 2 + \sqrt{3}$, evaluate $\sqrt{3}\cot(15^\circ)$
12. Using the triangle in Figure 2 and given $\alpha = 15^\circ 20'$ and $c = 3.59$, find $a$, $b$ and $\beta$. 

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13. Find the reference angle, $\theta_r$, for the following:
   
   a) $\theta = 120^\circ$
   
   b) $\theta = 315^\circ$
   
   c) $\theta = -150^\circ$
   
   d) $\theta = -328.2^\circ$
   
   e) $\theta = 1646.3^\circ$

14. For each of the following determine the reference angle, $\theta_r$, and evaluate the corresponding function:

   a) $\sin(225^\circ)$
   
   b) $\cos(570^\circ)$
   
   c) $\tan(-1500^\circ)$

1 Chapter 1

1. (§1.1) Complementary and supplementary angles:

   a) Complementary: Two angles that sum to $90^\circ$
   
   b) Supplementary: Two angles that sum to $180^\circ$

2. (§1.1) Convert between degrees/minutes/seconds (DMS) and decimal degrees

1.1 Example

Convert $44.0805434^\circ$ to DMS

- Convert the fraction of a degree to minutes:
  
  \[ (0.0805434 \text{ degrees})(60 \text{ minutes/1 degree}) = 4.8326' \]  

- Convert the fraction of a minutes to seconds
  
  \[ (0.8326 \text{ minutes})(60 \text{ seconds/1 minute}) = 49.956" \]  

- Bring everything together:
  
  \[ = 44^\circ 4' 49.956" \]
1.2 Example

Convert \(44^\circ 4' 49.956''\) to decimal degrees

Convert the seconds to minutes:

\[
\text{seconds to minutes:} \quad 49.956 \text{ seconds} = (49.956 \text{ seconds})(1 \text{ minute}/60 \text{ seconds}) = 0.8326' + 4' = 4.8326'
\] (4)

Convert the minutes to degrees:

\[
\text{minutes to degrees:} \quad 4.8326 \text{ minutes} = (4.8326 \text{ minutes})(1 \text{ degree}/60 \text{ minutes}) = 0.08054^\circ + 44^\circ = 44.08054^\circ
\] (5)

1.3 Example

Convert \(-103^\circ 13' 51.6''\) to decimal degrees

Convert seconds to minutes:

\[
\text{seconds to minutes:} \quad (51.6 \text{ seconds})(1 \text{ minute}/60 \text{ seconds}) = 0.86' + 13' = 13.86'
\] (6)

Convert minutes to degrees:

\[
\text{minutes to degrees:} \quad (13.86 \text{ minutes})(1 \text{ degree}/60 \text{ minutes}) = 0.231^\circ + 103^\circ = -103.231^\circ
\] (7)

3. (§1.1) Coterminal angles

1.4 Example

Find the coterminal angle whose measure is between \(-180^\circ\) and \(+180^\circ\) for each of the following.

1. \(\theta = 495^\circ\)

2. \(\theta = 645^\circ\)

1. \(\theta = 495^\circ\)

\[
495^\circ - 360^\circ = 135^\circ
\] (8)

135° satisfies the condition that the coterminal angle must be between \(-180^\circ\) and \(+180^\circ\).
1.5 Example

2. \( \theta = 645^\circ \)

\[
645^\circ - 360^\circ = 285^\circ
\] (9)

\(285^\circ\) is not between \(-180^\circ\) and \(+180^\circ\), so must continue:

\[
285^\circ - 360^\circ = -75^\circ
\] (10)

\(-75^\circ\) satisfies the condition that the coterminal angle must be between \(-180^\circ\) and \(+180^\circ\).

3. (§1.1) Special triangles. MEMORIZE the properties of 30-60-90 and 45-45-90 triangles
   a) Be able to find lengths of of unknown sides (see Example 4 and Problems 45-60)

1.5 Example

For a 30-60-90 triangle, if the hypotenuse \( h \) is of length 10, find the lengths of the other sides.

Knowing the ratios of sides in a 30-60-90 is \( 1x : \sqrt{3}x : 2x \) (where \( 2x \) is the length of the hypotenuse) we can find the value of \( x \) from the length of the hypotenuse:

\[
2x = 10 \rightarrow x = 5
\] (11)

Knowing \( x \), we can now substitute back into the ratio to find the lengths of the other sides:

\[
1x = 1(5) = 5
\] (12)

\[
\sqrt{3}x = \sqrt{3}(5) = 5\sqrt{3}
\] (13)

1.6 Example

For a 30-60-90 triangle, if the longer leg \( b \) measures \( 4\sqrt{3} \), find the lengths of the other sides.

Following a similar direction as in Example 1.2, we know the ratio of sides for a 30-60-90 triangle is: \( 1x : \sqrt{3}x : 2x \). Since the length of the longer leg (thus opposite the 60° angle) is specified, we can find the following:

\[
\sqrt{3}x = 4\sqrt{3} \rightarrow x = 4
\] (14)

Knowing \( x \), we can now find the length of side \( a \) and the hypotenuse:

\[
a = 1x = 1(4) = 4
\] (15)

\[
hyp = 2x = 2(4) = 8
\] (16)

4. (§1.2) Similar triangles
   a) Finding the unknown measure of a triangle
1.7 Example

Given the following angles of a right triangle, \( \angle A = 3x \), \( \angle B = (5x - 3) \), \( \angle C = 47 \), find the value of \( x \).

\[
A + B + C = 180 \quad (17)
\]

\[
3x + (5x - 3) + 47 = 180 \quad (18)
\]

\[
x = 17, \therefore A = 3(17) = 51, \quad B = 82 \quad (19)
\]

5. (§1.2) Similar triangles: Two triangles are similar if corresponding angles are equal: \( \triangle ABC = \triangle DEF \) if \( \angle A = \angle D \), \( \angle B = \angle E \), \( \angle C = \angle F \).

1.8 Example

Using Figure 3 find expressions for all sides of all triangles shown (four triangles in total).

Based on the triangle given in Figure 3 we can expand it as shown in Figure 4:

\( x \) is part of a 45-45-90 triangle, which has the ratio \( 1x : 1x : \sqrt{2}x \). Therefore:
\[ \sqrt{3} = \sqrt{2}x \rightarrow x = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2} \] 

(20)

\((y\) is also part of a 45-45-90:)

\[ 1 = \sqrt{2}y \rightarrow y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \] 

(21)

Now solve for the remaining lengths:

\[ x + y = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{2} \] 

(22)

\[ x - y = \frac{\sqrt{6} - \sqrt{2}}{2} \] 

(23)

6. (§1.3) Given a point, P on the terminal side of an angle, \(\theta\) find the value of the six trigonometric functions.

1.9 Example

Given \(\sin \theta = -\frac{3}{5}, \cot \theta < 0\), find the values of \(x, y\) and \(r\). Clearly indicate the quadrant of the terminal side of \(\theta\), then state the values of the six trig functions of \(\theta\).

\[ \sin \theta = -\frac{3}{5}, \cot \theta < 0 \] 

(24)

\[ y = -3, r = 5 \] 

(25)

Since \(\sin \theta < 0\), then we know we must be in QII or QIV. \(\cot \theta < 0 \) (\(\cot \theta = \frac{\text{adjacent}}{\text{opposite}}\)) is only possible in QII or QIV. Therefore, can only satisfy both conditions if in QIV.

\[ x = \sqrt{5 - (-3)^2} = 4 \] 

(26)

| \(\sin \theta\) | -\(\frac{3}{5}\) | \(\csc \theta\) | -\(\frac{5}{3}\) |
| \(\cos \theta\) | \(\frac{4}{5}\) | \(\sec \theta\) | \(\frac{5}{4}\) |
| \(\tan \theta\) | -\(\frac{3}{4}\) | \(\cot \theta\) | -\(\frac{4}{3}\) |

7. (§1.4) Identities. MEMORIZE the identities on page 32.

<table>
<thead>
<tr>
<th>Reciprocal Identities</th>
<th>Ratio Identities</th>
<th>Pythagorean Identities</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin \theta = 1/\csc \theta)</td>
<td>(\tan \theta = \sin \theta/\cos \theta)</td>
<td>(\sin^2 \theta + \cos^2 \theta = 1)</td>
</tr>
<tr>
<td>(\cos \theta = 1/\sec \theta)</td>
<td>(\tan \theta = \sec \theta/\csc \theta)</td>
<td>(\tan^2 \theta + 1 = \sec^2 \theta)</td>
</tr>
<tr>
<td>(\tan \theta = 1/\cot \theta)</td>
<td>(\cot \theta = \cos \theta/\sin \theta)</td>
<td>(1 + \cot^2 \theta = \csc^2 \theta)</td>
</tr>
</tbody>
</table>
2 Chapter 2

1. (§2.1) The majority of this section should be very familiar by now. However, pay special attention to Cofunctions and Complements, in §2.1-C.

Remember: complementary angles sum to 90°. Therefore, in a right triangle, α and β must sum to 90° since the total of the internal angles must sum to 180°. Mathematically,

\[ \alpha + \beta = 90^\circ \] (27)

From equation \([27]\):

\[ \beta = 90 - \alpha \] (28)

and

\[ \alpha = 90 - \beta \] (29)

Using the following triangle:

![Diagram](Figure 5: Cofunctions)

We can show that

\[ \sin(\alpha) = \frac{a}{c} \] (30)

and

\[ \cos(\beta) = \frac{a}{c} \] (31)

Thus,

\[ \sin(\alpha) = \cos(\beta) \] (32)

or

\[ \sin(\alpha) = \cos(90 - \alpha) \] (33)
2.1 Example

Find the value of \( x \) that makes \( \cos(2x) = \sin(3x) \).

\[ \alpha = 2x, \beta = 3x \]  
(34)

\[ \cos(\alpha) = \sin(90 - \alpha) \]  
(35)

\[ \beta = 90 - \alpha \therefore 3x = 90 - 2x \]  
(36)

\[ 3x + 2x = 90 \therefore x = 18 \]  
(37)

2.2 Example

If \( \tan(75^\circ) = 2 + \sqrt{3} \), evaluate \( \sqrt{3}\cot(15^\circ) \)

\[ \cot(\alpha) = \tan(90 - \alpha) \leftrightarrow \tan(\alpha) = \cot(90 - \alpha) \]  
(38)

\[ \sqrt{3}\cot(15^\circ) = \sqrt{3}\tan(90 - 15) = \sqrt{3}\tan(75) \]  
(39)

\[ = \sqrt{3}(2 + \sqrt{3}) = 2\sqrt{3} + 3 \]  
(40)

2. (§2.2) Be able to solve a right triangle given one angle and one side, and also given two sides. To solve a triangle means to find all angles and all sides given a minimum amount of information.

2.3 Example

Using the triangle in Figure 6 and given \( \alpha = 15^\circ20' \) and \( c = 3.59 \), find \( a \), \( b \) and \( \beta \).

To make things slightly easier to work with, first convert \( \alpha \) from DMS to decimal degrees:
Knowing the internal sum of the angles of a triangle = 180°:

\[ 180 = 90 + \alpha + \beta \rightarrow 90^\circ - 15.33^\circ = 74.67^\circ \] (42)

Given \( c = 3.59 \), we can use several different methods to solve for \( a \) and \( b \). We will proceed as follows:

\[ \sin(\alpha) = \frac{a}{c} \rightarrow 3.59 \cdot \sin(15.33^\circ) = 0.95 \] (43)

Knowing two of three sides, we can proceed using the Pythagorean Theorem:

\[ a^2 + b^2 = c^2 \] (44)

\[ b = \sqrt{(3.59)^2 - (0.95)^2} = \sqrt{11.986} = 3.46 \] (45)

3. (§2.4) Understand reference angles and how they allow you to extend beyond acute angles. Reference angles allow us to evaluate trigonometric functions for any angle. It’s important to understand that to find a reference angle you

a) First draw a vertical line between the terminal side and the \( x \)-axis; the angle formed between the terminal side and the \( x \)-axis becomes your reference angle.

b) Find the value of the trig function (e.g. sine, cosine, tangent, etc.) for the reference angle.

c) Perform a quadrant analysis to determine the proper sign.

### 2.4 Example

Find the reference angle, \( \theta_r \), for the following:

1. \( \theta = 120^\circ \)
   \( \theta_r = 60^\circ \)
2. \( \theta = 315^\circ \)
   \( \theta_r = 45^\circ \)
3. \( \theta = -150^\circ \)
   \( \theta_r = 30^\circ \) (Remember, reference angles are always positive)
4. \( \theta = -328.2^\circ \)
   \( \theta_r = 31.8^\circ \)
5. \( \theta = 1646.3^\circ \)
   \( \theta_r = 26.3^\circ \)
2.5 Example

For each of the following determine the reference angle, $\theta_r$, and evaluate the corresponding trigonometric function:

1. $\sin(225^\circ)$
   
   \[ \theta_r = 45^\circ \]  
   \[ \sin(45) = \frac{\sqrt{2}}{2} \]

   Performing a quadrant analysis we see $225^\circ$ is in QIII, giving a negative sine:
   
   \[ \sin(225) = -\frac{\sqrt{2}}{2} \]

2. $\cos(570^\circ)$
   
   \[ \theta_r = 30 \]
   \[ \cos(30) = \frac{\sqrt{3}}{2} \]

   Performing a quadrant analysis we see $570^\circ$ is in QIII, giving a negative cosine:
   
   \[ \cos(570) = -\frac{\sqrt{3}}{2} \]

3. $\tan(-1500^\circ)$
   
   \[ \theta_r = +60^\circ \]
   \[ \tan(60) = \frac{\sqrt{3}}{1} = \sqrt{3} \]

   Performing a quadrant analysis we see $-1500^\circ$ is in QIV, giving a negative tangent:
   
   \[ \tan(-1500^\circ) = -\sqrt{3} \]