1. Find all solutions in \([0, 2\pi)\) for the equation: 
\[-4\cos(x) = 2\sqrt{3}\]

\[-4\cos(x) = 2\sqrt{3} \quad (1)\]

\[\cos(x) = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2} \quad (2)\]

\[x = \frac{5\pi}{6} \quad (3)\]

\[x = \frac{7\pi}{6} \quad (4)\]

2. Find the principal root of the equation: 
\[-2\sqrt{3}\sin(2x) = -3\]

\[-2\sqrt{3}\sin(2x) = -3 \quad (5)\]

\[\sin(2x) = \frac{3}{2\sqrt{3}} \quad (6)\]

\[= \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} \quad (7)\]

Perform u-substitution:

\[u = 2x \quad (8)\]

\[\sin(u) = \frac{\sqrt{3}}{2} \quad (9)\]

\[u = \frac{\pi}{3} \quad (10)\]

Solve for \(x\):

\[u = 2x = \frac{\pi}{3} \rightarrow x = \frac{\pi}{6} \quad (11)\]
3. Find all real solutions for the equation: $2\sqrt{3}\tan(3x) = 6$

HINTS: Remember the period for tangent! In what quadrants is tangent positive?

EXTRA CREDIT: Explain/show why we really only have one equation as the solution for $x$.

\[
2\sqrt{3}\tan(3x) = 6 \quad (12)
\]
\[
\tan(3x) = \frac{3}{\sqrt{3}} = \sqrt{3} \quad (13)
\]

Perform u-substitution:

\[
u = 3x \quad (14)
\]
\[
\tan(u) = \sqrt{3} \quad (15)
\]

Remember, tangent is \(\text{opposite} \div \text{adjacent} = \frac{\sqrt{3}}{1}\); therefore, tangent is POSITIVE in both QI and QIII. Also, note the period for tangent is $\pi$!

\[
u = \frac{\pi}{3} + \pi k \quad (16)
\]
\[
u = \frac{4\pi}{3} + \pi k \quad (17)
\]

Solve for $x$:

\[
u = 3x = \frac{\pi}{3} + \pi k \rightarrow x = \frac{\pi}{9} + \frac{\pi}{3}k \quad (18)
\]
\[
u = 3x = \frac{4\pi}{3} + \pi k \rightarrow x = \frac{4\pi}{9} + \frac{\pi}{3}k \quad (19)
\]

EXTRA CREDIT:

Note that our solutions are coterminal. You can see this in either equations 16 and 17 or equations 18 and 19. In other words, $\frac{\pi}{9} + \pi = \frac{4\pi}{3}$ (equations 16 and 17) and $\frac{\pi}{9} + \frac{\pi}{3} = \frac{4\pi}{9}$ (equations 18 and 19). Thus, we can say the solutions are simply:

\[
x = \frac{\pi}{9} + \frac{\pi}{3} \quad (20)
\]