Supplementary Problems on Line Integrals

Determine whether $\mathbf{F}(x, y)$ is conservative. If so find a potential function $f(x, y)$.

1. $\mathbf{F}(x, y) = 9x^2y^2\mathbf{i} + (6x^3y - 1)\mathbf{j}$

2. $\mathbf{F}(x, y) = xy\mathbf{i}$

3. $\mathbf{F}(x, y) = \cos x \sin y\mathbf{i} + \sin x \cos y\mathbf{j}$

4. $\mathbf{F}(x, y) = 2xy\mathbf{i} + (x^2 + y^2)\mathbf{j}$

5. $\mathbf{F}(x, y) = e^x \sin y\mathbf{i} + e^x \cos y\mathbf{j}$

6-10. Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{c}$ for each of the functions in problems 1-5, where $C$ is the path

   a. counterclockwise along the semi-circle $y = \sqrt{1 - x^2}$ from $(1, 0)$ to $(-1, 0)$, then along the line segment from $(-1, 0)$ to $(2, 1)$.

   b. from $(1, 0)$ to $(2, 1)$ along the parabola $y = (x - 1)^2$.

   Hint: if $\mathbf{F}$ is conservative, problems 6-10 can be done with a very small amount of computation. If not, you have a bit of work to do.

Answers

1. Conservative; $f(x, y) = 3x^3y^2 - y$

2. Not conservative

3. Conservative; $f(x, y) = \sin x \sin y$

4. Conservative; $f(x, y) = x^2y + \frac{1}{3}y^3$

5. Conservative; $f(x, y) = e^x \sin y$

6ab. 23

7a. $\frac{3}{2}$

7b. $\frac{7}{12}$

8ab. $\sin 2 \sin 1 \approx 0.7651$

9ab. $\frac{13}{3}$

10ab. $e^2 \sin 1 \approx 6.218$