1. Solve the spherical heat equation

\[ \frac{\partial u}{\partial t} = k \nabla^2 u \]

subject to

\[ \frac{\partial u}{\partial \rho}(a, \theta, \phi, t) = 0 \]

for \( 0 < \rho < a, -\pi < \theta < \pi, \) and \( 0 < \phi < \pi. \)

Also, I made a mistake in the notes when we solved the spherical wave equation. When I indexed the eigenvalues of the spherical Bessel Equation, I used the index \( m, \) but that index had already been used. I should have indexed the eigenvalues with a new index, \( \lambda_{nl}, \) which would also create a third summation (a summation over \( l \)) in the final answer. If you have questions, come see me.

2. Use finite differences to approximate the solution to

\[ \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \]

subject to

\[ \frac{\partial u}{\partial x}(0, t) = -0.1u(0, t) \quad \text{and} \quad u(1, t) = 0 \]

with

\[ u(x, 0) = \begin{cases} 
0 & 0 \leq x < 0.4 \\
0.5 & 0.4 \leq x \leq 0.6 \\
0 & 0.6 < x \leq 1 
\end{cases} \quad \text{and} \quad \frac{\partial u}{\partial t}(x, 0) = \begin{cases} 
0 & 0 \leq x < 0.4 \\
-0.1 & 0.4 \leq x \leq 0.6 \\
0 & 0.6 < x \leq 1 
\end{cases} \]

Your submission should include your work for developing your finite difference scheme, as well as an output of \( u(x, 2) \) using \( c = 1, \Delta x = 0.1, \) and \( \Delta t = 0.1. \)

3. Use finite differences to approximate the solution to

\[ \frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u \]

for \( 1 < r < 2 \) and \( 0 < \theta < \pi \) subject to \( u(r, \theta, t) = 0 \) on the boundary with

\[ u(r, \theta, 0) = \begin{cases} 
0.5 & 1.4 \leq r \leq 1.6, \ \frac{\pi}{3} \leq \theta \leq \frac{\pi}{2} \\
0 & \text{otherwise} 
\end{cases} \quad \text{and} \quad \frac{\partial u}{\partial t}(r, \theta, 0) = \begin{cases} 
-1 & 1.45 \leq r \leq 1.55, \ \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3} \\
0 & \text{otherwise} 
\end{cases} \]

Your submission should include your work for developing your finite difference scheme, as well as an output of \( u(r, \theta, 20) \) using \( c = 1, \Delta r = 0.1, \) and \( \Delta \theta = \frac{\pi}{20}. \)
4. Use finite differences to approximate the solution to

\[
\frac{\partial u}{\partial t} = k \nabla^2 u + Q(r, \theta, t)
\]

for \(1 < r < 2\) and \(-\pi < \theta < \pi\) subject to \(\frac{\partial u}{\partial r}(1, \theta, t) = 0\) and \(u(2, \theta, t) = 0.1 \sin(t/5) \cos(2\theta)\) on the boundary with \(u(r, \theta, 0) = \max\{1 - \frac{1}{2}(r^2 + r \cos \theta), 0\}\), where

\[
Q = \begin{cases} 
-0.2 \sin(\theta + 0.5) \sin(2\pi r) \sin(\pi t/5) & t \geq 15, \ 1 < r \leq 3/2, \ -\pi/2 \leq \theta \leq \pi/2 \\
0 & \text{otherwise}
\end{cases}
\]

Your submission should include your work for developing your finite difference scheme, as well as an output of \(u(r, \theta, 20)\) using \(k = 0.01\), \(s = 0.25\), \(\Delta r = 0.1\), and \(\Delta \theta = \pi/20\).