Use Maple to row reduce the augmented matrix:
\[
\begin{pmatrix}
1 & -\alpha & b_2 \\
0 & 2-\alpha^2 & 1-\alpha^2 & b_2 - b_1 \alpha \\
0 & 0 & p(\alpha) & g(\alpha, b)
\end{pmatrix}
\]

Set \( p(\alpha) = 2 - 2\alpha^2 - \alpha + \alpha^3 = 0 \) to obtain \( \alpha = 1, \alpha = -1, \alpha = 2 \), the values that make \( A \) singular. Substitute these \( \alpha \) values into \( g \): \( g(1, b) = b_3 - b_1 \), so \( A\hat{x} = \hat{b} \) has a solution if and only if \( b_3 - b_1 = 0 \). Similarly, for \( \alpha = -1 \), we obtain \( b_3 + 3b_1 + 2b_2 = 0 \), and for \( \alpha = 2 \), we get \( b_2 - 3b_3 = 0 \).

1.5.15 If \( \text{rank} \ A = n \) then \( A\hat{x} = \hat{0} \) has at most one solution.

Equivalently, if \( A\hat{x} = \hat{0} \) has more than one solution, then \( \text{rank} \ A < n \).

a) \( A \hat{0} = \hat{0} \) and \( A \begin{pmatrix}
1 \\
0 \\
\vdots \\
0
\end{pmatrix} = \hat{0} \).

b) \( A \hat{0} = \hat{0} \) and \( A \begin{pmatrix}
c_1 \\
c_2 \\
\vdots \\
c_n
\end{pmatrix} = \hat{0} \). Since \( c_i \neq 0 \) for some \( i \), we have two distinct solutions to \( A\hat{x} = \hat{0} \).