1. A company produces two products. Relevant information for each product is shown in Table 1. The company has a goal of $48 in profits and incurs a $1 penalty for each dollar it falls short of this goal. A total of 32 hours of labor are available, a $2 penalty is incurred for each hour of overtime (labor over 32 hours) used, and a $1 penalty is incurred for each hour of available labor that is unused. Marketing considerations require that at least 10 units of Product 2 be produced. For each unit by which production falls short of demand a penalty of $5 is assessed.

(a) Formulate an LP that can be used to minimize the penalty incurred by the company.

(b) Suppose the company sets (in order of importance) the following goals.
Goal 1: Avoid underutilization of labor.
Goal 2: Meet demand for Product 2.
Goal 3: Do not use overtime.

Formulate and solve a preemptive goal programming model for this situation.

<table>
<thead>
<tr>
<th>Labor required</th>
<th>Product 1</th>
<th>Product 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution to profit</td>
<td>$4</td>
<td>$2</td>
</tr>
</tbody>
</table>

Table 1: Product information.
2. Mondo produces motorcycles at three plants. At each plant, the labor, raw material, and production costs (excluding labor costs) required to build a motorcycle are as shown in Table 2. Each plant has sufficient machine capacity to produce up to 750 motorcycles per week. Each of Mondo’s workers can work up to 40 hours per week and are paid $12.50 per hour worked. Mondo has a total of 525 workers and now owns 9,400 units of raw material. Each week, at least 1,400 Mondo’s must be produced. Let $x_i$ be the number of motorcycles produced at plant $i$. The goal is to minimize cost.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Labor Needed (hours)</th>
<th>Raw Material Needed (units)</th>
<th>Production Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>7</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2: Production information.

(a) The LP for this problem is given by:

$$
\text{min } z = 300x_1 + 280x_2 + 225x_3
$$

subject to:

$$
20x_1 + 16x_2 + 10x_3 \leq 21000
$$

$$
5x_1 + 8x_2 + 7x_3 \leq 9400
$$

$$
x_1 \leq 750
$$

$$
x_2 \leq 750
$$

$$
x_3 \leq 750
$$

$$
x_1 + x_2 + x_3 \geq 1400
$$

$$
x_1, x_2, x_3 \geq 0
$$

Use LINDO to solve this LP. Then use the sensitivity analysis to answer the following questions. Print out the formulation and sensitivity analysis reports and include it with your homework.

(b) What would be the new optimal solution to the problem if the production cost at plant 1 were only $40?

(c) How much money would Mondo save if the capacity of plant 3 were increased by 100 motorcycles?

(d) By how much would Mondo’s cost increase if it had to produce one more motorcycle?

(e) Suppose that Mondo could purchase an additional unit of raw material at a cost of $6. Should the company do it? Explain.
3. Steelco uses coal, iron, and labor to produce three types of steel. The inputs (and sales price) for one ton of each type of steel are shown in Table 3. Up to 200 tons of coal can be purchased at a price of $10 per ton. Up to 60 tons of iron can be purchased at $8 per ton, and up to 100 labor hours can be purchased at $5 per hour. Let \( x_i \) be the tons of steel \( i \) produced. The goal is to maximize profit.

<table>
<thead>
<tr>
<th>Steel</th>
<th>Coal Required (tons)</th>
<th>Iron Required (tons)</th>
<th>Labor Required (hours)</th>
<th>Sales Price ($/*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>51</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 3: Steel production information.

(a) The LP for this problem is given by:

\[
\begin{align*}
\text{max } z &= 8x_1 + 5x_2 + 2x_3 \\
\text{subject to: } & \\
3x_1 + 2x_2 + x_3 & \leq 200 \\
x_1 + x_3 & \leq 60 \\
x_1 + x_2 + x_3 & \leq 100 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]

Use LINDO to solve this LP. Then use the sensitivity analysis to answer the following questions. Print out the formulation and sensitivity analysis reports and include it with your homework.

(b) What would profit be if only 40 tons of iron could be purchased?

(c) What is the smallest price per ton for steel 3 that would make it desirable to produce it?

(d) Find the new optimal solution if steel 1 sold for $55 per ton.

(e) What is the most that Steelco should be willing to pay for an extra ton of coal?

(f) What is the most that Steelco should be willing to pay for an extra ton of iron?

(g) What is the most that Steelco should be willing to pay for an extra hour of labor?