Physics 213L: “University Physics Laboratory”
The Simple Pendulum
With an introduction to curve fitting using the method of least squares

Pre-lab Exercise

1. A simple pendulum of length 1.4 m makes 8 complete oscillations in 19 s. What is the acceleration due to gravity at this location?

2. What is the length of a simple pendulum that passes through the equilibrium position once each second?

3. What is minimized in a least squares algorithm?
Goal: This lab helps students: (1) understand the harmonic motion, (2) apply the formula to describe the motion of a simple pendulum, and (3) analysis and present the experiment data using the least square method.

Note: For a thorough discussion of the method of least squares, read pages provided with lab exercise.

1. Introduction:
Simple harmonic motion (SHM) results when the force that tends to restore a system to equilibrium is proportional to the displacement of the system from equilibrium: $F \propto s$.

Consider the simple pendulum diagrammed on the left. The bob of mass $m$ is suspended by a cord of length $L$. The dimensions of the mass must be small compared the length of the cord. The displacement of the pendulum from equilibrium is $s$. The magnitude of the restoring force $F$ is given by $F = m \cdot g \cdot \sin \theta$. For small angles $\sin \theta \sim \theta$. In addition, we may write $\theta = s/L$. Thus, the restoring force has the form $F = (mg/L)s$, and the condition for SHM is met.

Applying Newton’s second law in the $s$ direction yields:

$$-m \cdot g \cdot \sin \theta = m \cdot \left(\frac{d^2 s}{dt^2}\right).$$

The negative sign results because the displacement and the restoring force are in opposite directions. Since $s = L \theta$, we substitute $d^2 s/dt^2 = L d^2 \theta/dt^2$ and $\sin \theta \sim \theta$ into the above equation and find:

$$\frac{d^2 \theta}{dt^2} + \left(\frac{g}{L}\right) \cdot \theta = 0.$$

This is the standard SHM differential equation with solution $\theta(t) = \theta_0 \cdot \sin(\omega t + \phi)$, where $\omega = \sqrt{g/L}$ is the angular frequency of the harmonic motion and $\phi$ is the phase angle which depends on initial conditions. The period of the motion is the time required to complete one full cycle and is given by:

$$T = 2\pi \cdot \sqrt{\frac{L}{g}}.$$

If we square both sides of the equation for the period we get

$$T^2 = \left(4 \cdot \frac{\pi^2}{g}\right) \cdot L.$$
Thus, if $T^2$ is plotted versus $L$, a straight line should result with slope equal to $4 \pi^2/g$.

2. **Procedure:**

2.1 **Timer operation**  
Use the stopwatch provided to time 30 complete swings of the pendulum. Divide this time by 30 to get the period $T$.

2.2 **Dependence of the period on mass**  
With a mass of 50 grams and a length of 80 cm, measure the period of motion of the pendulum by timing 10–30 complete swings of the pendulum (Remember to divide this time by the number of swings to get the period).

Repeat the experiment with a mass of 100 grams and the same 80 cm length.

**Question:** Is there a significant dependence of period on mass?

2.3 **Dependence of the period on length**  
With a mass of 50 grams or 100 grams, measure the period of motion of the pendulum for three lengths other than 80 cm.

Plot $T^2$ versus $L$ ($T^2$ on y axis, $L$ on x axis) along with the appropriate data point from part 1.

Determine $g$, the acceleration due to gravity, by hand, from the slope of the best straight line drawn through the data. If the data are not approximately linear, check your calculations for consistent units or other errors. An acceptable value for $g$ should be close to 980 cm/s$^2$ or 9.8 m/s$^2$.

2.4 **Using the method of least squares**  
Carry out a least squares calculation by hand using the simple pendulum data.
- Calculate $g$ from the slope.
- The relevant equations are on page 132 of *Experimentation, 3rd Edition*. The argument of the sum in equation 6-5 is equation 6-1 on page 131.
- For reference, a sample least squares calculation is attached below in Section 3.

Now use Excel on the computer to plot and graph the data. Make a trend line for the data and show the equation for it. Calculate $g$ from the slope. Print your graph.

3. **Least Squares Sample Calculation: an example**  

3.1 Example data and calculations:
3.2 Some details of the calculations:

Please use numbers given in the table above to see if you can repeat the numbers below.

\[ N = 4 \]

\[ m = \frac{[(4)*(94)-(33)*(10)]}{[(4)*(30)-(10)*(10)]} = 2.3 \]

\[ b = \frac{[(30)*(33)-(10)*(94)]}{[(4)*(30)-(10)*(10)]} = 2.5 \]

\[ Q = 0.30 \]

\[ S_y = \sqrt{\frac{0.30}{2}} = 0.387 \]

\[ S_m = 0.387 \times \{4/[(4)*(30)-(10)*(10)]\}^{1/2} = 0.173 \]

\[ S_b = 0.387 \times \{30/[(4)*(30)-(10)*(10)]\}^{1/2} = 0.474 \]

Thus:

\[ b = 2.5 \pm 0.47 \]

\[ m = 2.3 \pm 0.17 \]