ATTENTION

Please be extremely careful around this experiment, as it is very sensitive. Bumping the balance or the table it is on is to be avoided at all costs. Writing in your notebooks while they are on the table holding the balance can cause unwanted movement of the pendulum, and is to be avoided. As you can see we have even taken special measures to isolate the balance from the vibrations in the floor, which although imperceptible to us, are enough to make the experiment impossible. Be aware that the pendulum can take several hours to come to rest after being bumped or shaken. For this reason there are spring-loaded pins located on the bottom of the aluminum case for use in dampening the motion of the pendulum. They are tricky to use at best, and should not be relied upon to stop the pendulum completely, but can help in slowing extreme motion.

The alignment procedure can be very time-consuming. It has been done by your lab instructor. Therefore you will not need to move the balance, or turn any of the knobs or thumbscrews. If you think the balance needs adjustment, please ask your instructor.

The beryllium copper ribbon inside the tall tube supports significant weight and can break easily. If it breaks, your group will have to schedule a make-up day to do the lab, since after replacement the ribbon takes 24 hours to stretch to its final length.
PHYS 213L: University Physics Laboratory
Gravitational Torsion/Cavendish Experiment

Pre-lab Exercises:

1. Determine the order of magnitude of the gravitational force that you exert on another person 2m away. Compare this force with the Coulomb force generated by two electrons 2m apart.

2. Using Kepler’s third law, calculate the mass of the Sun using the period of the Earth’s orbit around, and the distance from the sun.

Discussion during the Lab:

1. How much the gravitational torsion experiment can be affected by the existence of students inside the lab room?

2. Will it improve the resolution in the measurement of gravitational constant G by letting all students leave the lab room and come back to take a final data reading after the torsion reaches its final equilibrium state? Can you estimate by how much it improves if it does?
Physics 213L: University Physics Laboratory
Gravitational Torsion/Cavendish Experiment

**Objective:** To experimentally derive the fundamental gravitational constant $G$ using the gravitational torsion balance.

1. **Background:**

   In 1797 Henry Cavendish began what was to become one of the most important experiments of the scientific age. His goal was to determine the density of the Earth. This was a much-desired quantity in 18th-century astronomy, since once the Earth's density was known, the densities of the Moon, Sun, and the other planets could be found from it. He accomplished this by using an apparatus called a gravitational torsion balance. In a gravitational torsion balance, the force imparted on an object by gravity is countered by the force imparted on the same object by a twisted wire. In this way an extremely small force can be controlled and calculated. It was using this torsion balance that Cavendish succeeded in measuring the density of the Earth, finding it’s average to be about 5.5 times that of water. Many years later, scientists were beginning to recognize the significance of the gravitational constant $G$. In Cavendish's time, $G$ did not have the importance among scientists that it has today; it was simply a proportionality constant in Newton's law. These scientists saw that Cavendish’s measurements could be used to isolate and find the value of this universal constant. Using Cavendish’s 75-year old results, they were able to calculate $G$ to an accuracy that was within 1% of the modern value of:

   \[
   6.67 \times 10^{-11} \text{Nm}^2/\text{Kg}^2
   \]

2. **Overview:**

Fig 1. Assembled Gravitational Torsion Balance.
The Gravitational Torsion Balance (Fig. 1) consists of a pendulum assembly consisting of two 38.3-gram masses (unseen inside the case) and a mirror suspended from a highly sensitive torsion ribbon, and two 1.5-kilogram masses that can be positioned as required. The Gravitational Torsion Balance is oriented so the force of gravity between the small masses and the earth is negated (the pendulum is nearly perfectly aligned vertically and horizontally). The large masses are brought near the smaller masses, and the gravitational force between them causes the pendulum assembly to twist. An optical lever, produced by a laser aimed at the mirror, is used to accurately measure the angular displacement of the mirror and the period of oscillation of the pendulum, thus allowing for the calculation of $G$.

The gravitational attraction between a 15 gram mass and a 1.5 kg mass when their centers are separated by a distance of approximately 46.5 mm (a situation similar to that of the Gravitational Torsion Balance) is about $7 \times 10^{-10}$ newtons. If this doesn’t seem like a small quantity to measure, consider that the weight of the small mass is more than two hundred million times this amount. The enormous strength of the Earth's attraction for the small masses, in comparison with their own attraction for the large masses, is what originally made the measurement of the gravitational constant such a difficult task. The torsion balance (invented by Charles Coulomb) provides a means of negating the otherwise overwhelming effects of the Earth's attraction in this experiment. It also provides a force delicate enough to counterbalance the tiny gravitational force that exists between the large and small masses.

### 3. Principle of the Experiment:

With the large masses in Position I in Fig. 3, the gravitational attraction, $F$, between each small mass ($m_2$) and its neighboring large mass ($m_1$) is given by the law of universal gravitation:

$$F = G \frac{m_1 m_2}{b^2} \quad (1.1)$$

where $b =$ the distance between the centers of the two masses (Figure 3).

The gravitational attraction between the two small masses and their neighboring large masses produces a
net torque on the system:

\[ \tau_{\text{grav}} = 2Fd \quad (1.2) \]

Where \( d \) is the length of the lever arm of the pendulum bob crosspiece (Figure 3).

Since the system is in equilibrium (holding still), the twisted torsion band must be supplying an equal and opposite torque. This torque \( (\tau_{\text{band}}) \) is equal to the torsion constant for the band \( (\kappa) \) times the angle through which it is twisted \( (\theta) \), or:

\[ \tau_{\text{band}} = -\kappa\theta \quad (1.3) \]

Combining equations 1.1, 1.2, and 1.3, and taking into account that \( \tau_{\text{grav}} = -\tau_{\text{band}} \) gives:

\[ \kappa\theta = \frac{2dGm_1m_2}{b^2} \]

Rearranging this equation gives an expression for \( G \):

\[ G = \frac{\kappa\theta b^2}{2dm_1m_2} \quad (1.4) \]

To determine the values of \( \theta \) and \( \kappa \) — the only unknowns in equation 1.4 — it is necessary to observe the oscillations of the small mass system when the equilibrium is disturbed. To disturb the system from its equilibrium position \( S_1 \), the swivel support is rotated so the large masses are moved to Position II. The system will then oscillate until it finally comes to rest at a new equilibrium position \( S_2 \) (Figure 4). Notice how the amplitude of the wave - or in this case the distance traveled - is decreasing while the period stays the same. Keep this in mind later when you are timing the period of the laser spot oscillations.

![Fig. 4 Origin of T and the oscillation of the small-mass system](image-url)
At the new equilibrium position $S_2$, the torsion wire will still be twisted through an angle $\theta$, but in the opposite direction of its twist in Position I, so the total change in angle is equal to $2\theta$ (Fig. 5). Taking into account that the angle is also doubled upon reflection from the mirror, and using trigonometry we see:

$$\Delta S = S_2 - S_1$$

$$4\theta = \frac{\Delta S}{L} \quad \text{or,}$$

$$\theta = \frac{\Delta S}{4L} \quad (1.5)$$

where $L$ will change for each station and has been measured by your instructor during setup.

The torsion constant can be determined by observing the period ($T$) of the oscillations, and then using the relation:

$$T^2 = \frac{4\pi^2 I}{\kappa} \quad (1.6)$$

where $I$ is the moment of inertia of the small mass system. The moment of inertia for the mirror and support system for the small masses is negligibly small compared to that of the masses themselves, so the total inertia can be expressed as:

$$I = 2m_2\left(d^2 + \frac{2}{5}r^2\right) \quad (1.7)$$

Therefore:

$$\kappa = \frac{8\pi^2 m_2 \left(d^2 + \frac{2}{5}r^2\right)}{T^2} \quad (1.8)$$

Substituting equations 1.5 and 1.8 into equation 1.4 gives:
All the variables on the right side of equation 1.9 are known or measurable, which will allow you to determine \( G \).

\[
G = \frac{\pi^2 \Delta S b^2 \left( d^2 + \frac{2}{5} r^2 \right)}{T^2 m_1 L d}
\]  \hspace{1cm} (1.9)

(1.9)

4. Experiment Procedure:

(1) Turn on the laser and observe the Position I end point of the balance for several minutes to be sure the system is at equilibrium. Record the Position I end point \( (S_1) \) as accurately as possible, and indicate any variation over time as part of your margin of error in the measurement.

(2) Carefully rotate the swivel support so that the large masses are moved to Position II. The spheres should be just touching the case, but take care to avoid knocking the case or sliding the balance and disturbing the system.

(3) Use a stopwatch to determine the time required for one period of oscillation \( (T) \). For greater accuracy, include several periods, and then find the average time required for one period of oscillation. The spot will move slowly to the side and may stop for a few seconds before it starts moving back the other way. This is normal. Use your best judgment as to when to say the spot turned around. Expect periods to be between 7-10 minutes long. **Note:** The accuracy of this period value \( T \) is very important, since the \( T \) is squared in the calculation of \( G \).

(4) Wait until the oscillations stop (usually 60-90 minutes), and record the resting equilibrium point \( (S_2) \). Calculate \( \Delta S \).

(5) Use your results and equation 1.9 to determine a preliminary value of \( G \).

(6) The value calculated in step 2 is subject to the following systematic error:

The small sphere is attracted not only to its neighboring large sphere, but also to the more distant large sphere, though with a much smaller force. The geometry for this second
force is shown in Figure 6 (the vector arrows shown are not proportional to the actual forces).

From Figure 6

\[ f = F_0 \sin \Phi \]

\[ \sin \Phi = \frac{b}{(b^2 + 4d^2)^{1/2}} \]

The force, \( F_0 \), is given by the gravitational law, which translates, in this case, to:

\[ F_0 = \frac{G_0 m_1 m_2}{b^2 + 4d^2} \]

and has a component \( f \) that is opposite to the direction of the force \( F \):

\[ f = \frac{G_0 m_1 m_2 b}{(b^2 + 4d^2)^{3/2}} \]

This equation defines a dimensionless parameter, \( R \) that is equal to the ratio of the magnitude of \( f \) to that of \( F \) (see Fig. 6), i.e. \( R = \frac{f}{F} \). Using the equation \( F = \frac{G_0 m_1 m_2}{b^2} \), it can be determined that:

\[ R = \frac{b^3}{(b^2 + 4d^2)^{3/2}} \]

From Fig. 6,

\[ F_{net} = F - f = F - RF = F (1 - R) \]

where \( F_{net} \) is the value of the force acting on each small sphere from both large masses, and \( F \) is the force of attraction to the nearest large mass only.

Similarly,

\[ G = G_0 (1 - R) \]

in which \( G \) is your experimentally determined value for the gravitational constant, and \( G_0 \) is corrected to account for the systematic error.

Finally,
\[ G_0 = \frac{G}{1 - R} \]

(7) Use this equation with equation 1.9 to adjust your measured value.

(8) Compare the gravitational constant value you get with the value in your textbook. Calculate the percentage of the difference between them.