Problem 1. A first order heterogeneous irreversible reaction (A → B) is taking place within a spherical catalyst pellet which is coated with Pt throughout the pellet. The reactant concentration halfway between the external surface and the center of the pellet (r=R/2) is equal to 1/10th the concentration of the pellet’s external surface. The concentration at the external surface is 0.001 gmol/dm³, the diameter (2R) is 0.002 cm, and the diffusion coefficient is 0.1 cm²/s.

a) What is the concentration of the reactant at a distance of 3 x 10⁻⁴ cm in from the external pellet surface?

b) To what diameter should the pellet be reduced if the effectiveness factor is to be 0.8?

Problem 2. Assume a steady state shell balance on a spherical catalyst pellet and derive a differential equation of the form:

\[
\frac{d^2 \psi}{d \lambda^2} + \frac{2}{\lambda} \left( \frac{d \psi}{d \lambda} \right) - \phi^2 \psi^n = 0
\]

describing both diffusion and reaction. Further show that for a first order reaction, the dimensional concentration profile reduces to:

\[
\psi = \frac{C_A}{C_{A_s}} = \frac{1}{\lambda} \left( \frac{\sinh \phi \lambda}{\sinh \phi_1} \right)
\]

where \( \psi = \frac{C_A}{C_{A_s}} \), \( \phi \) = Thiele modulus, \( \lambda = \frac{r}{R} \)

Further show that the internal effectiveness factor is related to Thiele modulus by the following relationship

\[
\eta = 3\phi_1^{-2} \left( \coth \phi_1 - 1 \right)
\]