6.5.1 Unforced Responses and Fundamental Properties of Natural Modes

In this section we investigate the unforced response properties of the systems shown in Figure 6.5-1. These exercises are designed to engender an intuitive understanding of the physical behavior of natural modes of oscillation and their associated frequencies. Such a hands-on physical exposure is of great benefit in understanding the corresponding analytical concepts.

![Setup for Configuration "a"

Third carriage not used

Medium stiffness springs (nominally 450 N/m)

Setup for Configuration "b"

Third carriage not used

Securing brace weights as per procedure instructions (Model 210A shown)

Figure 6.5-2. Mechanism Setup for Experiments in Section 6.5

Procedure: Free-Free Boundary Conditions

1. Setup the mechanism as shown in Figure 6.5-2 with two large brass weights and one small brass weight secured to the first carriage. Assuming that the larger weights have mass 0.500 kg, and the smaller one has mass 0.250 kg, use the results of Section 6.1 to determine the number of large weights required for the second carriage such that its mass approximates that of the first. Secure these weights to the second carriage. This is now configuration "a" in Figure 6.5-1. Determine the mass for each carriage.

2. In the following procedural steps, avoid contacting the limit switches with the mass carriages – particularly when the carriage has significant velocity. The resulting impact could damage the switch or the stop.

With the control box powered off, manually move the first carriage. Start with the carriage near its limit of travel on the side of the drive motor. Move it slowly at first but then rapidly accelerate it in a way as to induce minimal vibration between the carriages and to release it soon enough that there is free travel before the carriages contact the limit stops. Repeat this process if necessary and observe the resulting motion. This is the rigid body mode; so called because in the ideal case there is no relative motion between the inertias and hence they behave as if they were rigidly coupled. The motion after your release of the carriage approximates the unforced response of the system to a non zero initial
velocity of the rigid body modal coordinate. Set up and acquire encoder 1 and 2 data during the period immediately after release.¹

3. Manually oscillate the first carriage approximately two cycles per second. You should feel a frequency at which the system “naturally” tends to oscillate. How much force is required to sustain the oscillations when applied at this natural frequency? Excite the system at a higher frequency (roughly harmonic) and then at a lower frequency than the natural frequency. How much force is required to sustain these oscillations?

Practice exciting the system such that a minimum amount of drift (rigid body motion) occurs after release. Zero the encoder outputs. Again acquire encoder 1 and 2 data during the period immediately after release. Observe the physical behavior of the system and then the plotted data. (You may wish to zoom the plot to view only several cycles of motion when there is minimal drift of the carriages). Disregarding any drift in the output, what is the direction and amplitude of $x_1$ relative to $x_2$ at any given time? What is the frequency of oscillation and how does it relate to the parameters $m_1, m_2,$ and $k$?

4. Again excite the rigid body and oscillatory modes as per Steps 2 and 3 (no need to acquire data) except this time manipulate the second carriage. How does the change in location of applied force affect the mode shapes and frequencies?

5. Determine the number of brass weights for the second carriage such that $m_2 = \frac{1}{2}m_1$ (approximately). Secure this number of weights to the second carriage. Repeat Step 3. How does changing one inertia affect the natural frequency and mode shape? Based on these results and Eq’s (5.2-8, -9) what is the approximate frequency and oscillatory mode shape for $m_1 \gg m_2$?

Clamped-Free Boundary Conditions

6. Secure the same number of weights to the second carriage as determined in Step 1. Secure the second medium stiffness spring between the first carriage and the spring bracket as per Figure 6.5-2b. Hence configure the mechanism as per Figure 6.5-1b.

7. Gently attempt to accelerate the first carriage as in Step 1. Does a rigid body mode exist for this configuration?

8. Repeat the test procedure in Step 3 with an initial oscillation frequency of roughly 1 Hz. Repeat again attempting to find a higher frequency mode. How many oscillatory modes exist? What are the mode shapes (relative magnitudes and directions of $x_1$ and $x_2$) for each case?

9. Zero the encoder outputs. Refer to your plotted data from Step 8. Displace the first and second carriage according to the first mode shape measured in that step. Use the encoder display on the Background Screen to ascertain the carriage displacements or use the built-in rulers. While collecting data² attempt to release the two carriages simultaneously.³ You will probably need several attempts to satisfactorily coordinate having both the proper modal displacement amplitude ratio (and signs) and simultaneous release of the carriages. Observe the system

¹ You will need to set up a fictitious input shape of zero amplitude and 5 - 10 second duration, then “Execute” to begin data sampling. You may ask a fellow student to select “Run” at the proper time as you manipulate the carriage.

² Here, you will definitely need a fellow student to select “Run” at the proper time.

³ A suggested technique for simultaneously releasing the carriages is as follows. Hold the upper/outer edge of each carriage using your fingertips (or fingernails). Raise your fingers of each hand simultaneously to release the carriages.
and the resulting plot.1 What is the predominant shape or nature of the resulting free motion? What would the shape be under ideal conditions? Repeat this process for the second mode shape.

10. Repeat Step 9 except this time displace the system initially according to \( X(0) = [0 \ 3.0 \text{ cm}]^T \) (where \(^T\) denotes the transpose) and with zero initial velocity. Can you see the contribution of each mode to the system response? May the unforced system response to arbitrary initial conditions be described solely in terms of superposition of the modes? Assuming the motion attributable to each mode shape varies in time according to \( A_i \sin (\omega t + \psi_i) \) where \( i \) is the mode number and \( A_i \) and \( \psi_i \) are constants — what is the form of the system response and what do \( A_i \) and \( \psi_i \) depend on?

6.5.2 Driven Response: Impulse and Step Inputs

In this section we investigate the response of the systems shown in Figure 6.5-2 to impulse and step inputs. These discontinuous input responses are commonly used both analytically and in industrial practice to characterize the system.

Procedure (Free-Free Boundary Conditions)

1. Setup the mechanism as shown in Figure 6.5-2a with sufficient brass weights such that \( m_1 = m_2 \) (i.e. identical to the first configuration in the previous experiment). This is now configuration "a" in Figure 6.5-1. Position the first carriage near its limit of travel in the direction of the drive motor. Enable the force (torque) driving function.

2. With data sampling every 1 servo cycle, execute a unidirectional impulse of 15 N amplitude 25 ms pulse width and 1000 ms dwell. Plot the Encoder (Encoders 1 & 2) and Drive Input data. What are the salient characteristics of the response shape over the first one-half second of motion? How do these relate to the mode measurement results of the previous experiment? Which carriage is first to respond to the impulse? Why?

3. Remove all brass weights from the first carriage, zero the encoder values, and repeat the procedure of Step 2. How does the response differ from that of Step 2? Why?

4. Replace and secure the brass weights on the first carriage that were removed in Step 3 and remove all brass weights from the second carriage. Zero the encoder values, and repeat the procedure of Step 2. How does the response differ from those of Steps 2 and 3? Why? Assume that the inertias of the first and second mass carriages with the weights removed are equal. By swapping the names of the output variables \( x_1 \) and \( x_2 \) relative to their physical locations, the systems in Steps 3 and 4 are then identical. Thus, consider Step 4 as exciting the same system as Step 3 except at a different input location (i.e. at the larger inertia in Step 4 and at the smaller inertia in Step 3.) How does forcing function input location effect the shape of the output?

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1 You may want to plot velocity rather than position data because the former is less sensitive to low frequency drift effects. You should be able to explain why the velocity amplitude ratios are the same as the position ratios in this harmonically oscillating case.
5. Replace and secure the brass weights on the second carriage so that the system is in the original configuration of Step 1. Repeat the procedure of Step 2 except execute a Step input of 2 N for 500 ms. How does the motion compare with the results of Step 2?

Procedure (Clamped Compliance - Free Boundary Conditions)

6. Repeat Step 6 of Experiment 6.5.1. Hence set up the system as per Figure 6.5-1b.

7. Repeat the procedure of Step 2 except increase the dwell time to 2000 ms and the impulse force to 20 N. What are the salient characteristics of the response shape over the first two seconds of motion? How does this compare with the results of Step 2? How does it relate to the results of the previous section?

8. Remove all brass weights from the second carriage. Repeat the procedure of Step 2. How does the motion compare with that observed in Step 7?

9. Replace and secure the brass weights on the second carriage. Repeat the procedure of Step 2 except execute a Step input of 8 N for 2 second. How does the motion compare with the results of Steps 5 and 7?

6.5.3 Driven Response: Harmonic Input

In this section we investigate the response of the systems shown in Figure 6.5-2 to harmonic inputs. These inputs allow us to measure important system characteristics such as resonant and anti-resonant frequencies, input-to-output phase, and mode shapes.

Procedure (Free-Free Boundary Conditions)

1. Setup the mechanism as shown in Figure 6.5-2a with sufficient brass weights such that \( m_1 = m_2 \) (i.e. identical to the first configuration in the previous experiment). This is now configuration "a" in Figure 6.5-1.

2. Setup the Force + Spring + Damper driving function with \( k = 80 \text{ N/m and } \epsilon = 2.0 \text{ N/m/s} \) via the Setup Driving Function dialog box.\(^1\) With the first carriage approximately centered within its travel range, Enable the driving function. Setup a logarithmic sine sweep from 0.8 to 10 Hz, 1.2 N amplitude, and 120 second sweep time. Setup to acquire Encoder 1 Position, Encoder 2 Position and Drive Input data once every 2 servo cycles.

Important Note: Whenever coefficients \( k \) or \( \epsilon \) are utilized via the Force + Spring + Damper driving function, the user should never apply excessive force (> approx. 5 N) or force for a prolonged duration (> approx. 5 sec) to the mechanism. Since these parameters are effected via the servo motor and amplifier, any such high force or long duration could cause excessive heating and result in damage to the equipment.


\(^{1}\) Here as in Section 6.4.1 we employ a small amount of spring force and damping to obtain data without undue drift and to help stabilize the control system. Can you show that these unmodeled parameters do not qualitatively effect the results through the frequency spectrum tested?
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4. Execute the sine sweep maneuver. Watch the system closely during the sweep (you may repeat the sweep if necessary). What is the relative magnitude of oscillation at low frequencies? Do the masses move in phase with each other at these frequencies? As frequency increases what happens to oscillation amplitude? Is there a frequency at which the first carriage becomes nearly motionless? As frequency increases further does the system experience a resonance? If so, are the carriages in phase or 180° out of phase with each other? At yet higher frequencies, what happens to the oscillation amplitude? Does the first carriage amplitude change more or less rapidly than the second carriage with increasing frequency at these high frequencies? Plot the data using Logarithmic Frequency - Db scaling with Remove DC bias selected. Review the amplitude-frequency dependence discussed above on your plotted data. Note the slope of the traces for each encoder at low and high frequency. Save your plot. If the centerline of the response plot wanders excessively during the maneuver you may wish to run the maneuver a second time and/or use the techniques described in Section 6.4.1.

5. Replot the data, including Drive Input, under linear time-amplitude scaling to view the phase between the drive input and the encoders. You may wish to do this for each encoder separately. Zoom the plot to view the phase between the input and the encoders at low frequency (say 1 Hz), in the frequency neighborhood of the first carriage anti-resonance (low amplitude at approx. 2.3 Hz), in the neighborhood of the resonance, and at high frequency. Use the technique described in Section 6.4.1 to approximate the phase in each of these cases for both encoder outputs.

6. Identify the resonant frequency, \( \omega_r \), from the sweep of Step 4. Reduce the Drive Input amplitude to 1.0 N and perform a 60 second sine sweep of -10% to +10% about \( \omega_r \). From this data, identify \( \omega_r \) more precisely and perform an additional sine sweep of -1% to +1% about \( \omega_r \). Plot the resulting data (linear time-amplitude scaling, both encoders plotted on the same axis) and measure the frequency (no. of cycles / \( \Delta t \)) where the amplitude peaks. How does this frequency relate to the natural frequency for this system as studied in Section 6.5.1? Zoom in the plot to view several cycles about this peak. How does this motion compare with the mode shapes measured in Section 6.4.1?

Procedure (Clamped Compliance - Free Boundary Conditions)

7. Repeat Step 6 of Experiment 6.5.1. Hence set up the system as per Figure 6.5-1b. Make certain that the spring attachment bracket is positioned so that the carriages are in their approximate center of travel. Enable the force (torque) driving function.

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1 You may ignore the amplitude of the first several cycles which are unrepresentatively low due to the startup transient.
2 Encoder data on left axis and Drive input on right axis.
3 Several factors should be considered when identifying the resonant frequency. First, you should consider the second mass amplitude more than the first in evaluating the resonant frequency. This is because the first mass amplitude is affected by its lower frequency anti-resonance and tends to exhibit maximum amplitude at frequencies slightly higher than the second mass. Second, recall that due to damping, the resonant frequency is slightly higher than the natural frequency. Third, because time is required for the system to attain maximum amplitude at a fixed frequency (approximately 20 sec in this case), the indicated maximum during the frequency sweep will occur at a higher indicated frequency than the maximum under ideal (infinitesimal sweep rate) conditions. Thus the sweep rate should be very slow when identifying the resonant frequency and in viewing the corresponding mode shapes.
8. Repeat the procedure of Step 4. Observe again the motion as frequency increases. How does the low frequency motion and Do amplitude slope compare at low and high frequency with that of Configuration "a" tested in Step 4? How many resonances and antiresonances are there in this case for each inertia? How does this compare with the results of Step 4? How does it relate to the results of the previous sections?

9. Repeat Step 5 and include an approximation of the phase behavior in the frequency neighborhood of each resonance and anti-resonance.

10. Repeat the procedure of Step 6 for each resonance. How do the frequencies and mode shapes compare with the measured results from Section 6.5.1?

11. Remove the brass weights from the second carriage. Repeat Step 4. Comparing the result with that of Step 8, how does lowering the second carriage mass affect the frequency response? What is a physical explanation?

12. Repeat Step 6. How do the mode shapes compare with those from Step 10? What is a physical explanation?