Continuous Compounding Derivation

We know previously that \( \lambda = \left(1 + \frac{r}{m}\right)^m - 1 \).

As compounding becomes more frequent, as moving from yearly to semi-annual to quarterly to monthly to weekly to daily to hourly to continuous, we can explore \( m \to \infty \).

\[
\lim_{m \to \infty} \lambda = \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^m - 1
\]

By definition \( \lim_{h \to \infty} \left(1 + \frac{1}{h}\right)^h = e = 2.71828 + \).

If we let \( \frac{r}{m} = \frac{1}{h} \), or \( m = rh \),

then \( \lambda = \lim_{h \to \infty} \left(1 + \frac{r}{rh}\right)^{rh} - 1 \)

\[ \lambda = \lim_{h \to \infty} \left(1 + \frac{1}{h}\right)^{hr} - 1 \]

\[ \lambda = \lim_{h \to \infty} \left[\left(1 + \frac{1}{h}\right)^h\right]^r - 1 \]

\[ \lambda = e^r - 1 \] Hooray!

Aside: If we have \( \lim_{m \to \infty} \) and since \( m = rh \),

then \( \lim_{m \to \infty} = \lim_{rh \to \infty} \). Since \( r \) is a finite value the \( \lim_{m \to \infty} = \lim_{h \to \infty} \).